Abdelhafid Boussouf University - Mila
Faculty of Science and Technology
Common Core ST Department
Module: Analysis 1

Academic Year: 2025/2026
1st Year Common Core ST
And 1st Year Engineering
Semester I

$$\boxed{\text{Analysis I: Tutorial Exercise Sheet 5}}$$

Exercise 01:

1. Using the definition of the definite integral (by Riemann sums), approximate $\int_1^2 \frac{dx}{x}$ and give a geometric interpretation of the result.

باستخدام تعريف التكامل المحدد (بمجموع ريمان)، قرب $\int_1^2 \frac{dx}{x}$ وأعط تفسيرًا هندسيًا للنتيجة.

2. Show how the approximation in part (1) can be improved. (. (1) وضح كيف يمكن تحسين التقريب في الجزء)

Exercise 02

Prove the following (constants of integration are omitted): (:أثبت ما يلي (ثوابت التكامل محذوفة)

1. $\int u^p \, du = \frac{u^{p+1}}{p+1}, \ p \neq -1.$

2. $\int \frac{du}{u} = \ln u.$

3. $\int (u+v) \, dx = \int u \, dx + \int v \, dx.$

4. $\int \cos u \, du = \sin u.$

Exercise 03

Find the following integrals. (.أوجد التكاملات التالية)

(1) $\int x\sqrt{x^2+1}\,dx$ (2) $\int x^2 e^{x^3} \cos(e^{x^3})\,dx$ (3) $\int \frac{1-\cos x}{x-\sin x}\,dx$ (4) $\int \frac{x^3}{\sqrt{1-x^4}}\,dx$ (5) $\int e^x \sin(e^x)\,dx$

Exercise 4: Integration by Parts

(a) Prove the formula for integration by parts: $\int u \, dv = uv - \int v \, du$ (برهن صيغة التكامل بالتجزئة)

by starting from the product rule for differentiation. (.إنطلاقا من قاعدة الضرب في الإشتقاق)

(b) Use the formula to find: $\int xe^{2x} \, dx$ (:استخدم الصيغة لإيجاد)

Exercise 5: Definite Integrals Using Substitution

Evaluate the following definite integrals: (احسب التكاملات المحددة التالية) (1) $\int_0^1 \frac{3x^2}{x^3+1}\,dx$ (2) $\int_0^{\pi/4} \sec^2(2x)\,dx$

Exercise 6: Area Under a Curve

Find the area under the curve $y = \cos x$ between $x = 0$ and $x = \pi/2$.

أوجد المساحة تحت المنحنى $y = \cos x$ بين $x = 0$ و $x = \pi/2$.

Exercise 7: Mixed Substitution and Definite Integral

1. Find $\int \dfrac{\ln x}{x}\, dx$ using an appropriate substitution. (أوجد $\int \dfrac{\ln x}{x}\, dx$ باستخدام تعويض مناسب.)

2. Evaluate $\int_0^1 x e^{-x^2}\, dx$. (احسب)

Exercise 8: Integration by Parts with Trigonometric Functions

Find: $\int x\cos(3x)\, dx$ (أوجد:)

Exercise 9: Definite Integral of a Rational Function

Evaluate: $\int_1^e \dfrac{dx}{x(1+\ln x)}$ (احسب:)

Exercise 10: Area Between Curves

Find the area enclosed between $y = x^2$ and $y = \sqrt{x}$ from $x = 0$ to $x = 1$.

أوجد المساحة المحصورة بين المنحنيين $y = x^2$ و $y = \sqrt{x}$ من $x = 0$ إلى $x = 1$.

Exercise 11: Proof and Application

1. Prove that $\int \sec^2 x\, dx = \tan x + C$. (برهن أن)

2. Use this result to evaluate $\int_0^{\pi/4} \sec^2 x\, dx$. (استخدم هذه النتيجة لحساب)

Exercise 12: (Integration of Rational Fractions)

Compute the following integrals: (أحسب التكاملات التالية:)

$$1)I_1 = \int \frac{2x-1}{(x-1)(x-2)}\, dx, \qquad 2)I_2 = \int \frac{x\, dx}{(x+1)(x+3)(x+5)}, \qquad 3)I_3 = \int \frac{x^5 + x^4 - 8}{x^3 - 4x}\, dx.$$

Exercise 13:

Approximate using Taylor series: $\int_0^1 \frac{1-e^{-x}}{x}\, dx$. (قرب باستعمال سلسلة تايلر)

Exercise 14:

Prove the following equalities: (أثبت العلاقات التالية:)

1. $\int \frac{x}{x^2-3x+2}\, dx = 2\ln(x-2) - \ln(x-1) + C$

2. $\int \frac{dx}{x^2+x-2} = \frac{1}{3}\ln\left|\frac{x-1}{x+2}\right| + C$

3. $\int \frac{dx}{(x-1)^2(x-2)} = \frac{1}{x-2} + \ln\left|\frac{(x-2)}{(x-1)}\right| + C$

4. $\int \frac{x-8}{x^3-4x^2+4x}\, dx = \frac{3}{x-2} + \ln\frac{(x-2)^2}{x^2} + C$

5. $\int \frac{x^5}{x^3-1}\, dx = \frac{1}{3}\left[x^3 + \ln|x^3 - 1|\right] + C$

6. $\int \frac{\arcsin x}{x^2}\, dx = \ln\left|\frac{1-\sqrt{1-x^2}}{x}\right| - \frac{1}{x}\arcsin(x) + C$