

**Material 4 : Duality**

**Exercise 1.** Consider the following linear program (P):

$$\text{Max } Z=3x_1-x_2$$

$$\begin{cases} 2x_1+x_2 \geq 2 \\ x_1+3x_2+x_3=3 \\ x_1+x_2 \geq 1 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

- Solve (P) geometrically.
- Write the dual (D) of (P).
- Solve (D) using the complementary slackness theorem.

**Exercise 2.** Consider the following linear program:

$$\text{Max } Z=3x_1+x_2+3x_3.$$

$$\begin{cases} 2x_1+x_2+x_3 \leq 2 \\ x_1+2x_2+3x_3 \leq 5 \\ 2x_1+2x_2+x_3 \leq 6 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

- Solve the program using the simplex method.
- Write the dual of this linear program and deduce an optimal solution for it.

**Exercise 3.** Consider the following linear program (P):

$$\text{Max } Z=4.5x_1+2.5x_2+x_3+3x_4$$

$$\begin{cases} 2x_1+x_2+x_4 \leq 3 \\ x_1-2x_2+2x_3 \leq 10 \\ 3x_1+2x_2+x_3+2x_4 \leq 8 \\ x_1, x_2, x_3, x_4 \geq 0 \end{cases}$$

The last table of the simplex is as follows.

VB	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	b
$x_4$	2	1	0	1	1	0	0	3
$x_6$	3	-2	0	0	4	1	-2	6
$x_3$	-1	0	1	0	-2	0	1	2
Z	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	-1	0	-1	-11

- Write the dual (D) of (P) and deduce its optimal solution from the last table of the primal.
- Suppose that  $C_4=4$  (the coefficient of  $x_4$  in Z). Give the optimal solutions of (P) and (D).