University of Mila | **Academic Year: 2025/2026**

Institute of Nature and life Sciences | **1^{st} Year NLS, Semester 1**

Department of NLS | **Mathematics Statistics**

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Exercises Serie N° 3

Exercise 1

Determine the domain of definition of each of the following functions:

❶ $f(x,y) = \dfrac{xy}{x^2+y^2+1}$ ❷ $f(x,y) = \sqrt{\dfrac{x^2-y}{y}}$

❸ $f(x,y) = x^2 + y + \ln(x^2+y^2)$ ❹ $f(x,y) = e^{x-\frac{1}{y}}$

Exercise 2

❶ Calculate the partial derivatives of order 1 of the following functions:

a) $f(x,y) = (x^2+y^2)^4$ **b)** $f(x,y) = \arctan\frac{y}{x}$

c) $f(x,y) = \sqrt{4-xy}\,\sin\left(\frac{x}{y}\right)$ **d)** $f(x,y) = \ln(x+\sqrt{x^2+y^2})$

❷ Calculate the gradient of f at the point P

$$f(x,y) = e^y\cos(3x+y), \qquad P = \left(\frac{2\pi}{3},\,0\right)$$

Exercise 3

Calculate the partial derivatives of order 2 of the following functions:

❶ $f(x,y) = \dfrac{\ln(y^2+1)}{x}$ ❷ $f(x,y) = \sqrt{1+x^2+y^2}$

❸ $f(x,y) = x^4 + y^3 + 2y\cos x + 5y$

Exercise 4

Calculate the following double integrals:

❶ $I_1 = \iint\limits_D (x+y)\,\sin(x)\sin(y)\,dx\,dy$ $D = [0,\pi]\times[0,\pi]$

❷ $I_2 = \iint\limits_D \frac{x^2}{y}\,dx\,dy$ $D = [-1,1]\times[1,2]$

❸ $I_3 = \iint\limits_D yx^2 \, dx \, dy$

With:

$$D = \{(x, y) \in \mathbb{R}^2 \mid x \leqslant 1,\ y \geqslant 0,\ y^2 \leqslant x\}$$

❹ $I_4 = \iint\limits_D xye^{y-x} \, dx \, dy$

With:

$$D = \{(x, y) \in \mathbb{R}^2 \mid x \leqslant 0,\ y > 0,\ y - x < 1\}$$

Exercise 5

Calculate the following triple integrals:

❶ $I_1 = \iiint\limits_P x^2 y e^{xyz} \, dx \, dy \, dz$

With:

$$P = [0, 1] \times [0, 2] \times [-1, 1]$$

❷ $I_2 = \iiint\limits_P y \, dx \, dy \, dz$

With:

$$P = \{(x, y, z) \in \mathbb{R}^3 \mid y \geqslant 0,\ x^2 + y^2 + z^2 \leqslant 1\}$$