

Practical Sessions: Series 3

■ Exercise 1:

One-Factor ANOVA (Single Factor)

Problem:

We want to compare the yield (in kg) of 3 wheat varieties (A, B, and C) grown under the same conditions. Each variety was tested on 4 plots (replications).

| Variety | Plot 1 | Plot 2 | Plot 3 | Plot 4 |
|---------|--------|--------|--------|--------|
| A | 20 | 22 | 19 | 21 |
| B | 25 | 27 | 26 | 28 |
| C | 18 | 17 | 19 | 20 |

Questions:

1. Compute the mean of each variety and the overall mean.
2. Compute the sum of squares between groups (SCEf) and residual sum of squares (SCEr).
3. Compute mean squares (MSf, MSr) and F statistic.
4. Test at $\alpha = 0.05$ whether the means differ significantly.
5. Complete the ANOVA table.

Excel Solution using Formulas:

Step 1: Means

- Formula in Excel to calculate row mean: =AVERAGE (B2 : E2) (for A), =AVERAGE (B3 : E3) (for B), etc.
- Overall mean: =AVERAGE (B2 : E4)

Step 2: Sum of Squares

- Between groups (SCEf): =SUMPRODUCT (COUNT (B2 : E2 : B4 : E4) , (AVERAGE (B2 : E2 : B4 : E4) - overall_mean) ^2)
- Within groups (SCEr): =SUMXMY2 (B2 : E2 , AVERAGE (B2 : E2)) + SUMXMY2 (B3 : E3 , AVERAGE (B3 : E3)) + SUMXMY2 (B4 : E4 , AVERAGE (B4 : E4))
- Total sum of squares: =SCEf + SCEr

Step 3: Mean Squares

- MSf: =SCEf / (P-1)
- MSr: =SCEr / (N-P)

- F statistic: $=MS_f/MS_r$

Step 4: F Test

- Compare F with $F.INV.RT(0.05, df_1, df_2)$ in Excel to determine significance.

Step 5: ANOVA Table in Excel:

| Source | df | SS | MS | F |
|----------------|----|-------|-------|------|
| Between Groups | 2 | 144.2 | 72.1 | 43.2 |
| Within Groups | 9 | 15 | 1.667 | |
| Total | 11 | 159.2 | | |

Conclusion: $F_{obs} = 43.2 > F_{critical} = 4.26 \rightarrow$ reject H_0 . Means are significantly different.

Two-Factor ANOVA (Factor A \times Factor B)

Problem:

Factor A: 3 levels (A1, A2, A3)

Factor B: 2 levels (B1, B2)

Each cell has $n = 4$ replications.

| | A1 | A2 | A3 |
|----|----------|-------------|-------------|
| B1 | 8,9,7,10 | 12,11,13,12 | 14,15,13,14 |
| B2 | 6,7,5,6 | 10,9,11,10 | 12,11,12,13 |

Excel Solution using Formulas:

Step 1: Cell Means

- `=AVERAGE(range of each cell)`

Step 2: Row and Column Means

- Row mean: `=AVERAGE(cell means in row)`
- Column mean: `=AVERAGE(cell means in column)`
- Grand mean: `=AVERAGE(all cell means)`

Step 3: Residual Variance

- `=SUMPRODUCT(n-1, variance of each cell)/(p*q*(n-1))`

Step 4: Sums of Squares

- Factor A: $SCE_A = n*q*\text{SUM}((\text{column_mean} - \text{grand_mean})^2)$
- Factor B: $SCE_B = n*p*\text{SUM}((\text{row_mean} - \text{grand_mean})^2)$
- Interaction: $SCE_{AB} = n*\text{SUM}((\text{cell_mean} - \text{row_mean} - \text{column_mean} + \text{grand_mean})^2)$

- Residual: $SCE_R = \text{SUM}((x_{ijk} - \text{cell_mean})^2)$

Step 5: ANOVA Table in Excel

| Source | df | SS | MS | F |
|-------------|----|--------|-------|-------|
| Factor A | 2 | 136.26 | 68.13 | 40.85 |
| Factor B | 1 | 28.25 | 28.25 | 16.95 |
| Interaction | 2 | 0.335 | 0.167 | 0.10 |
| Residual | 6 | 10 | 1.667 | - |
| Total | 11 | 174.85 | - | - |

Step 6: F Test

- Compare each F with critical F: $=F.INV.RT(0.05, df1, df2)$
- Factor A: $F_A > F_{critical} \rightarrow$ significant
- Factor B: $F_B > F_{critical} \rightarrow$ significant
- Interaction: $F_{AB} < F_{critical} \rightarrow$ not significant

Conclusion: Factors A and B are significant; interaction is not significant.