
Practical Sessions: Series 2

■ Exercise 1: Small Sample (Student t Test)

Problem:

Measure nitrate concentration (mg/kg) in 10 soil samples:

12.5, 13.2, 11.8, 12.9, 13.0, 12.4, 11.9, 12.6, 12.7, 12.3

Tasks:

1. Compute sample mean and standard deviation.
2. Compute the standard error (SE).
3. Determine the 95% confidence interval for the mean.
4. Interpret the result.

Step 0: Enter the data

In **Column A**, enter your 10 soil sample values:

A

12.5

13.2

11.8

12.9

13.0

12.4

11.9

12.6

12.7

12.3

Step 1: Sample mean

In any empty cell, e.g., **B1**, type:

=AVERAGE (A1 : A10)

- This will give **12.53**
-

Step 2: Sample variance and standard deviation

- **Sample variance (using your formula $S^2 = \Sigma(xi - \bar{x})^2 / n$):**

In **B2**, type:

```
=VAR.P(A1:A10)
```

- This gives **0.1841** (population variance formula, since n in denominator).

Note: Excel's VAR.P divides by n; VAR.S divides by n-1.

- **Sample standard deviation:**

In **B3**, type:

```
=SQRT(B2)
```

- This gives **0.429**
-

Step 3: Standard error (SE)

Using formula $SE = S / \sqrt{n-1}$

In **B4**, type:

```
=B3/SQRT(COUNT(A1:A10)-1)
```

- This gives **0.143**
-

Step 4: 95% Confidence Interval

- Degrees of freedom: $df=n-1=9$
- t-value for 95% CI (two-tailed) can be calculated in Excel using:

```
=T.INV.2T(0.05, COUNT(A1:A10)-1)
```

- In **B5**, type:

```
=T.INV.2T(0.05, COUNT(A1:A10)-1)
```

- This gives **2.262**
- **Margin of error:**

```
=B4*B5
```

- In **B6**, type:

```
=B4*B5
```

- Gives **0.323**
- **Lower CI:**

=B1-B6

- **Upper CI:**

=B1+B6

- Results:

CI Value

Lower 12.21

Upper 12.85

✓ Step 5: Summary in Excel Table

Statistic	Formula	Value
Mean ()	=AVERAGE (A1 : A10)	12.53
Variance (S ²)	=VAR . P (A1 : A10)	0.1841
Std. dev (S)	=SQRT (B2)	0.429
SE	=B3 / SQRT (COUNT (A1 : A10) - 1)	0.143
t (95% CI)	=T . INV . 2T (0 . 05 , COUNT (A1 : A10) - 1)	2.262
Margin of error	=B4 * B5	0.323
95% CI	Lower: =B1-B6 Upper: =B1+B6	[12.21, 12.85]

Exercise 2 : Large Sample (Grouped Data)

Problem:

A quality control test for 150 tablets; sodium bicarbonate content (mg):

Class	[1610,1615)	[1615,1620)	[1620,1625)	[1625,1630)	[1630,1635)
Frequency	7	8	42	75	18

Tasks:

1. Compute sample mean and standard deviation.
2. Compute the standard error (SE).
3. Determine the 95% confidence interval for the mean.
4. Interpret the result.

Step 0: Organize data

Class	Frequency (f)	Midpoint (x)
1610-1615	7	1612.5
1615-1620	8	1617.5
1620-1625	42	1622.5
1625-1630	75	1627.5
1630-1635	18	1632.5

Step 1: Sample mean

In Excel:

1. Column D: $f * x \rightarrow =B2*C2$
2. Sum of D: $=SUM(D2:D6)$
3. Sum of frequencies: $=SUM(B2:B6)$
4. Mean: $=SUM(D2:D6) / SUM(B2:B6)$

- **Result:**
 $\bar{x} = 1625.25$ mg
-

Step 2: Sample standard deviation

Formula for grouped data:

1. Column E: $(x - \text{mean})^2 \rightarrow =(C2 - \text{mean})^2$
2. Column F: $f * (x - \text{mean})^2 \rightarrow =B2*E2$
3. Sum of F: $=SUM(F2:F6)$
4. Variance: $=SUM(F2:F6) / SUM(B2:B6)$
5. Standard deviation: $=SQRT(\text{variance})$

- **Result:** $S \approx 4.79$ mg
-

Step 3: Standard error using $n-1$

- Total sample: $n = \sum f_i = 150$
- Formula in Excel:

$$= S / SQRT(SUM(B2:B6) - 1)$$

- **Result:**

$$SE \approx 4.79 / \sqrt{149} \approx 0.392$$

Step 4: 95% Confidence Interval

Use $Z = 1.96$ (large sample):

$$CI = \bar{x} \pm Z * SE$$

- Margin of error: $= 1.96 * SE \rightarrow \approx 0.769$
- Lower limit: $= \text{mean} - \text{margin} \rightarrow 1625.25 - 0.769 \approx \mathbf{1624.48}$
- Upper limit: $= \text{mean} + \text{margin} \rightarrow 1625.25 + 0.769 \approx \mathbf{1626.02}$

Step 5: Summary Table

Statistic	Value
Mean (\bar{x})	1625.25
Std. dev (S)	4.79
SE	0.392
95% CI	[1624.48, 1626.02]

✓ Interpretation:

With 95% confidence, the **true mean sodium bicarbonate content** is **between 1624.48 mg and 1626.02 mg**.

📊 Exercise 3 : One-Sample t-Test (Test of Conformity)

Objective: Test whether the mean soil organic matter content differs from the national reference value $m_0 = 4.5\%$.

Data:

Sample No.	1	2	3	4	5	6	7	8	9	10	11	12
Organic Matter (%)	4.2	4.8	5.0	4.1	4.4	4.7	4.3	4.9	4.5	4.6	4.0	4.8

Step 1: Enter Data

A

4.2

4.8

5.0

4.1

4.4

4.7

4.3

4.9

4.5

A

4.6

4.0

4.8

(Paste these in cells A1:A12)

Step 2: Compute Sample Statistics

Cell	Formula	Explanation
B1	=COUNT (A1 :A12)	Sample size n = 12
B2	=SUM (A1 :A12)	Sum of observations = 54.3
B3	=AVERAGE (A1 :A12)	Sample mean = 4.525
B4	=VAR . S (A1 :A12)	Sample variance = 0.09854
B5	=STDEV . S (A1 :A12)	Sample standard deviation = 0.3140
B6	=B5/SQRT (B1-1)	Standard error = 0.0946

Step 3: Compute t-Statistic

Cell	Formula	Explanation
B7	=B3-4.5	Difference from reference = 0.025
B8	=B7/B6	t statistic = 0.2641
B9	=B1-1	Degrees of freedom = 11

Step 4: Critical Value (Two-Tailed Test, $\alpha = 0.05$)

Cell	Formula	Explanation
B10	=T . INV . 2T (0.05, B9)	Two-tailed critical t \approx 2.201

Step 5: Decision

Cell	Formula	Explanation
B11	=IF (ABS (B8)>B10, "Reject H0", "Do not reject H0")	Result: "Do not reject H0"

■ Exercise 4:

Problem: We want to compare the average exam scores of two independent classes to determine whether there is a significant difference between their mean scores.

Data

- Class A ($n_1 = 8$): 78, 82, 85, 90, 76, 88, 84, 79
- Class B ($n_2 = 7$): 72, 75, 80, 77, 74, 79, 70

Significance level: $\alpha = 0.05$ (two-tailed)

Tasks:

1. Compute the sample means of sample 1 and 2.
2. Compute the sample variances of sample 1 and 2.
3. Perform a two-sample t-test to compare the two population means.

Step 1: Enter the Data

Class A	Class B
78	72
82	75
85	80
90	77
76	74
88	79
84	70
79	

(Class A in A1:A8, Class B in B1:B7)

Step 2: Compute Sample Means

- **Class A mean** \rightarrow =AVERAGE (A1:A8) \rightarrow **82.75**
 - **Class B mean** \rightarrow =AVERAGE (B1:B7) \rightarrow **75.29**
-

Step 3: Compute Sample Variances

- **Class A variance** \rightarrow =VAR.S (A1:A8) \rightarrow **20.935**
- **Class B variance** \rightarrow =VAR.S (B1:B7) \rightarrow **11.515**

Use VAR.S because these are **sample variances**.

Step 4: Compute Pooled Variance (Equal Variance Assumption)

1. **Weighted sum of squares:**

$$= (\text{COUNT}(A1:A8) - 1) * \text{VAR.S}(A1:A8) + (\text{COUNT}(B1:B7) - 1) * \text{VAR.S}(B1:B7) \rightarrow 248.083$$

2. **Pooled variance:**

$$= F1 / (\text{COUNT}(A1:A8) + \text{COUNT}(B1:B7) - 2) \rightarrow \mathbf{19.083}$$

3. **Standard error:**

$$= \text{SQRT}(F2 * (1 / \text{COUNT}(A1:A8) + 1 / \text{COUNT}(B1:B7))) \rightarrow \mathbf{2.260}$$

Step 5: Compute t-Statistic

$$t_{\text{obs}} = (\text{Mean}_A - \text{Mean}_B) / \text{SE} \rightarrow (D1 - D2) / F3 \rightarrow \mathbf{3.29}$$

Step 6: Degrees of Freedom and Critical Value

- **Degrees of freedom:** $= \text{COUNT}(A1:A8) + \text{COUNT}(B1:B7) - 2 \rightarrow \mathbf{13}$
 - **Critical t (two-tailed, $\alpha=0.05$):** $= \text{T.INV.2T}(0.05, H1) \rightarrow \mathbf{2.160}$
-

Step 7: Decision

- **Compare t_{obs} with t_{critical} :**
 $t_{\text{obs}} > t_{\text{critical}} \rightarrow \text{Reject } H_0 \rightarrow \mathbf{\text{There is a significant difference between the classes.}}$
-

Step 8: 95% Confidence Interval for Difference

- **Difference of means:** $= D1 - D2 \rightarrow 7.46$
- **Lower bound:** $= J1 - H2 * F3 \rightarrow 2.56$
- **Upper bound:** $= J1 + H2 * F3 \rightarrow 12.36$

✓ **CI:** [2.56, 12.36]