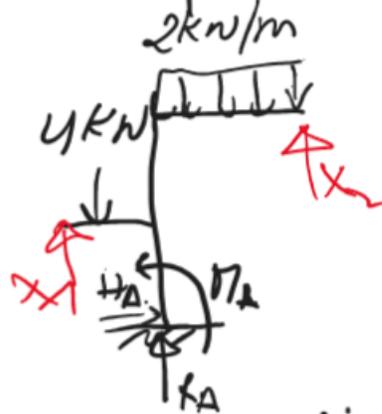Exercice 02

1) le degré d'hyperstaticité :

$$R = 3C - a - 2S$$
$$= 3(2) - 0 - 2(2) = 6 - 4 = 2.$$

2) structure isostatique équivalente (Système de base)
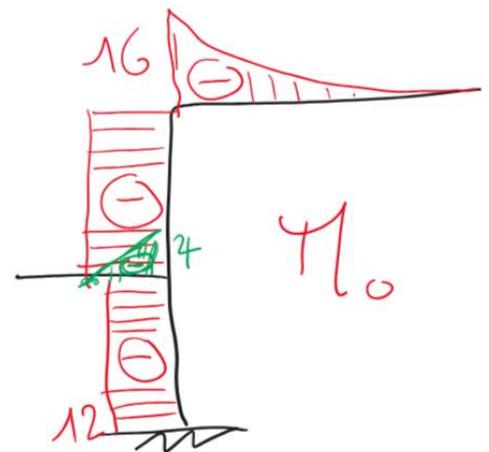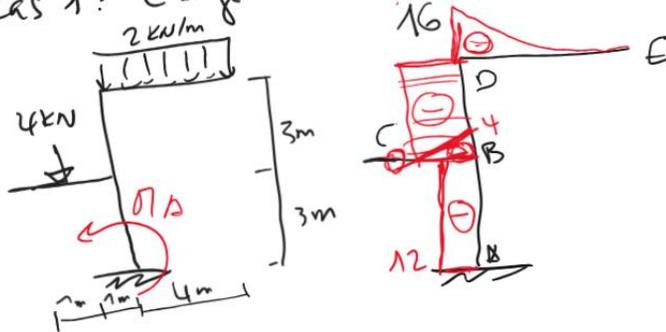

3) Système d'équation de compatibilité :

$$\begin{cases} \delta_{11} X_1 + \delta_{12} \cdot X_2 + \delta_{1f} = 0 \\ \delta_{21} X_1 + \delta_{22} X_2 + \delta_{2f} = 0 \end{cases}$$

4) Tracer les diagrammes de Moment fléchissant :

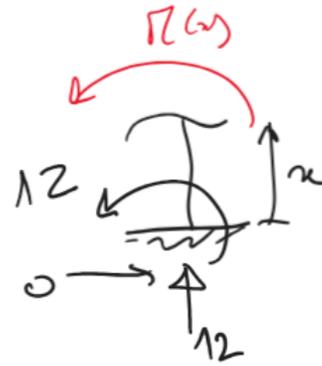4.1) Cas 1 : Charge ext ≠ 0, $X_1 = X_2 = 0$:

$\Sigma f_x = 0 \rightarrow H_A = 0$

$\Sigma f_y = 0 \rightarrow V_A = 12 \, kN$

$\Sigma M/_A = 0 \rightarrow M_A - 2(4) \cdot 2 + 4(1) = 0$

$M_A = 16 - 4 = 12$

* barre $AB : 0 \leq x \leq 3$:

$\circlearrowleft) \ \Sigma M(x)/_n = 0 \ \ M(x) + 12 = 0$

$M(x) = -12$

* barre $BD : 3 \leq x \leq 6$:

$\circlearrowleft) \ \Sigma M(x)/_n = 0 \rightarrow M(x) + 12 + 4(1) = 0$

$M(x) = -12 - 4 = -16$
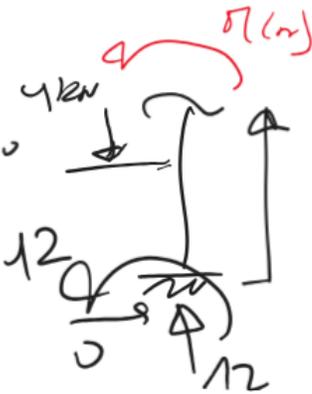
4.2) Cas 2 : charge ext $= 0$, $x_1 = 1$, $x_2 = 0$:

m_1

$x_1 = 1$
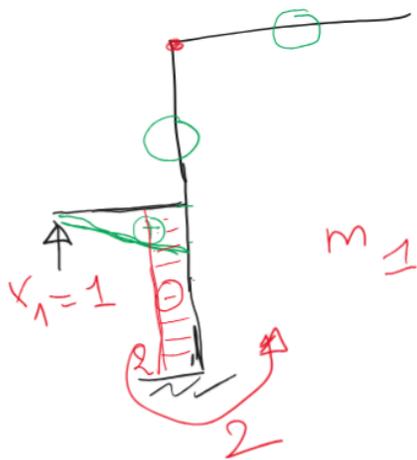
4.3) Cas 3: charge ext=0, $x_1=0$, $x_2=1$

$\sum M_{(n)} = 0$

$M(n) - 4 = 0$

$M(n) = 4$.
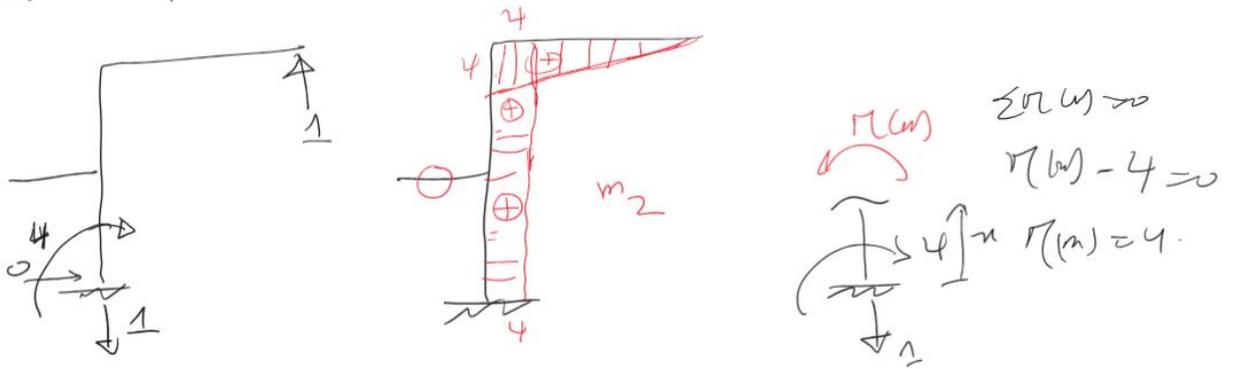
5) Calculer les coefficients δ_{ij} et δ_{iF}

$$\delta_{11} = \frac{44}{3EI} \quad , \quad \delta_{22} = \frac{352}{3EI} \quad , \quad \delta_{12} = \delta_{21} = \frac{-24}{EI}$$

$$\delta_{22} = \boxed{\triangleright} \times \boxed{\triangleright} + \boxed{\square} \cdot \boxed{\square}$$

$$\frac{1}{3} \cdot 4 \cdot 4 \cdot \frac{4}{EI} + 4 \cdot 4 \cdot \frac{6}{EI}$$

$$\delta_{22} = \frac{64}{3EI} + \frac{96}{EI} = \frac{352}{3EI} = 117,33$$

$$\ast \; \delta_{1F} = \boxed{4 \times} \; \boxed{2} \; \left(12 \boxed{-} \times \boxed{-} \right.$$

$$= \left(\frac{1}{6}(-4) \cdot (1 + 2(2)) \frac{1}{EI} \right) + \left((-12) \cdot (-2) \cdot \frac{3}{EI} \right)$$

$$= \left(-\frac{4}{6}(5) \frac{1}{EI} \right) + \left(\frac{72}{EI} \right) = -\frac{10}{3EI} + \frac{72}{EI}$$

$$\boxed{\delta_{1F} = \frac{206}{3EI}}$$

$$\ast \quad \delta_{2f} = \left(\overset{16}{\underset{4}{\triangle}} \times 4\boxed{\underset{+}{\overset{4}{\triangle}}}\right) + \left(16\boxed{\ominus} \times \boxed{\oplus 4}\right)$$

$$+ \left(12\boxed{\underline{}} \times \boxed{\oplus 4}\right)$$

$$\delta_{2f} = \left(\frac{1}{4}\cdot(-16)\cdot 4\cdot\frac{4}{EI}\right) + \left((-16)\cdot 4\cdot\frac{3}{EI}\right) + \left((-12)\cdot 4\cdot\frac{3}{EI}\right)$$

$$= \frac{-64}{EI} - \frac{192}{EI} - \frac{144}{EI} = \frac{-400}{EI}$$

6) Equation de compatibilité :

$$\begin{cases} \dfrac{44}{3EI}X_1 - \dfrac{24}{EI}X_2 + \dfrac{206}{3EI} = 0 \\[2mm] -\dfrac{24}{EI}X_1 + \dfrac{352}{3EI}X_2 - \dfrac{400}{EI} = 0 \end{cases} \Rightarrow \begin{cases} \dfrac{44}{3}X_1 - 24X_2 + \dfrac{206}{3} = 0 \\[2mm] -24X_1 + \dfrac{352}{3}X_2 - 400 = 0 \end{cases}$$

$$\begin{cases} 44X_1 - 72X_2 + 206 = 0 \\ -72X_1 + 352X_2 - 1200 = 0 \end{cases} \Rightarrow \begin{cases} 22X_1 - 36X_2 + 103 = 0 \\ -36X_1 + 176X_2 - 600 = 0 \end{cases}$$

$$\begin{cases} 792X_1 - 1296X_2 + 3708 = 0 \sim (1) \\ -792X_1 + 3872X_2 - 13200 = 0 \quad -(2) \end{cases}$$

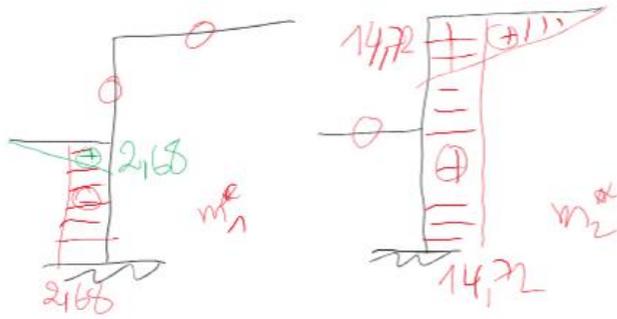$$(1)+(2) \rightarrow 2576\cdot X_2 - 9492 = 0$$
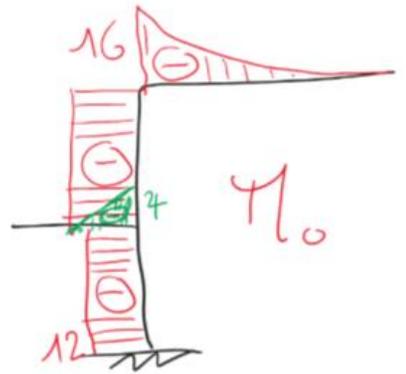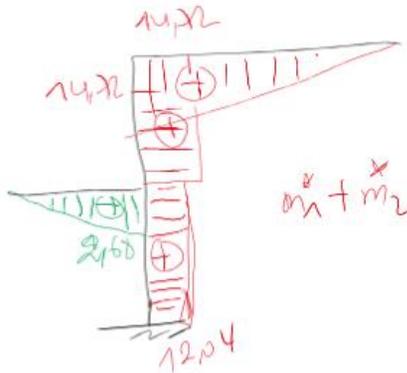
$$X_2 = \frac{9492}{2576} = 3,68 \text{ kN}$$

$$X_1 = 1,34 \text{ kN}$$

7) corriger les épures unitaires

$$m_1^{\ast} = m_1\cdot X_1, \quad m_2^{\ast} = m_2\cdot X_2$$

β) $m_1^a + m_2^a$

α) $M_{final} = M_0 + m_1^a + m_2^a$

$M_{max} = H(2,16) = 3,38\ kN\cdot m$

$\overset{?}{M}_{final}\ ?!$