

Chapter 4: Diffusion phenomena

1-Introduction

Diffusion is known as a means of exchanging solute molecules from one compartment to another across a membrane. It tends to uniform the distribution of particles (ions or non-dissociable molecules), occurring in the direction of decreasing concentrations, meaning the particles move from an area of high concentration to an area of low concentration.

2-Definition

The diffusion phenomenon across a semi-permeable membrane, under the influence of a concentration gradient, involves the movement of solute from the more concentrated medium to the less concentrated medium. Diffusion is the movement of solute particles.

Theoretical Aspect of the Diffusion Phenomenon - Fick's Law

Fick expressed the law that governs the diffusion phenomenon with an equation that aligns well with the one governing the heat propagation phenomenon in a conductor. It is assumed that:

- Diffusion occurs in a single direction in space.
- The concentration of the diffusing solute is constant across the entire x-axis plane.

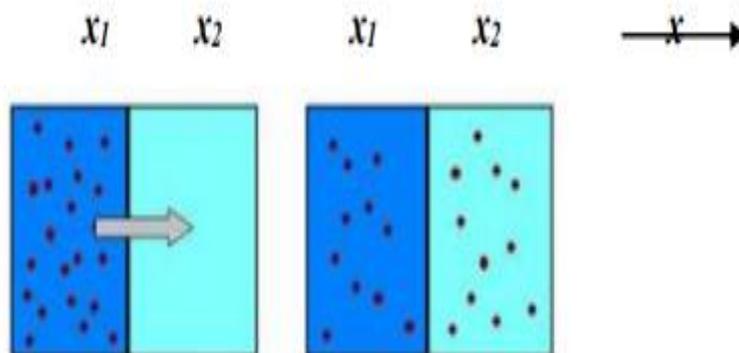


Fig: Diffusion phenomena.

Let Δm be the mass of the solute that diffuses over the time interval Δt from point x_1 to x_2 in the compartment

$$\Delta m = -D \cdot \frac{C_2 - C_1}{x_2 - x_1} \cdot \Delta t \cdot S$$

Thus

$$\Delta m = -D \cdot S \cdot \frac{\Delta C}{\Delta x} \cdot \Delta t \quad \Rightarrow \quad \frac{\Delta m}{\Delta t} = -D \frac{\Delta C}{\Delta x} S$$

Where:

$\frac{\Delta C}{\Delta x}$: Negative concentration gradient ($\Delta C < 0$, $\Delta x > 0$)

S: Cross-sectional area of the tube

D: Diffusion coefficient

Units:

Δm : kg; g;

ΔC : kg/m³; g/cm³; mole/m³; mole/cm³

Δx : m; cm

Δt : s

D: m²/s; cm²/s

$\frac{\Delta m}{\Delta t}$: Diffusion flux or the mass of solute that has moved during the time Δt from point

X_A with weight concentration C_A to point X_B with weight concentration C_B

It is also possible to express the number of moles diffusing (instead of the mass of molecules) per unit of time:

$$\frac{\Delta n}{\Delta t} = -D S \frac{\Delta c^M}{\Delta x}$$

$\frac{\Delta n}{\Delta t}$: Molar Flow Rate in moles/s.

The diffusion coefficient D depends on the experimental conditions of diffusion, such as temperature, the nature of the solute, and the nature of the solvent.

$$D = \frac{k_B T}{f}$$

Where

k_B is the Boltzmann constant ($1.38 \times 10^{-23} \text{J/K}$)

T is the temperature in Kelvin (K),

f is the friction coefficient, which is defined as follows: if the particle moves with velocity v in a liquid medium, it must overcome a frictional force F that is proportional to v :

$$F = f v$$

In the particular case where the particle is spherical with radius r , it can be shown (Stokes' law) that:

$$F = 6 \pi \eta r v$$

Thus, the friction coefficient is $f = 6\pi\eta r$

The diffusion coefficient D is proportional to the temperature T . Therefore, if the temperature and diffusion coefficient increase, the diffusion flux $\frac{\Delta m}{\Delta t}$ also increases.

Consider two solutes with radii r_1 and r_2 diffusing in two media with viscosities η_1 and η_2 , and two temperatures T_1 and T_2 , respectively.

We have:

$$D_1 = \frac{k T_1}{6 \pi \eta_1 r_1}, \quad D_2 = \frac{k T_2}{6 \pi \eta_2 r_2}$$

$$\frac{D_2}{D_1} = \frac{T_2 \eta_2 r_1}{T_1 \eta_1 r_2}$$

$$\Rightarrow D_2 = \frac{T_2 \eta_2 r_1}{T_1 \eta_1 r_2} D_1$$

Consider the case of two solutes, 1 and 2, which are different but diffuse under the same experimental conditions ($\eta_1=\eta_2, T_1=T_2$), so.

$$D_2 = \frac{r_1}{r_2} D_1$$

Consider the case where the volumetric density of the solute molecules 1 is equal to that of the 2nd solute ($\rho_1 \approx \rho_2$).

We have $\rho = \frac{m}{v}$ With $V = \frac{4}{3}\pi r^3$ (V: volume of a spherical molecule)

$$M = N m \quad \Rightarrow \quad \rho = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi r^3} \quad \Rightarrow \quad r = \sqrt[3]{\frac{M}{\frac{4}{3}\pi N \rho}}$$

We then obtain for the two solutes:

$$\frac{r_1}{r_2} = \sqrt[3]{\frac{M_1}{M_2}}$$

since $\rho_1 \approx \rho_2$.

Thus, for two solutes with molar masses M_1 and M_2 diffusing under the same experimental conditions, the diffusion coefficient D_2 is given by the following formula:

$$D_2 = \sqrt[3]{\frac{M_1}{M_2}} D_1$$