Introduction

Vibration is an oscillatory physical phenomenon of a body moving around its equilibrium position

- Among the most varied mechanical movements, there are movements which are repeated: the beating of the heart, the movement of a swing, the reciprocating movement of the pistons of an internal combustion engine.

All of these movements have one common trait: a repetition of the movement over a cycle.

- A cycle is an uninterrupted series of movements or phenomena which are always renewed in the same order .
- We call periodic movement a movement which repeats itself and each cycle of which reproduces itself identically. The duration of a cycle is called period.
- A particularly interesting periodic movement in the field of mechanics is that of an object which moves from its equilibrium position and returns to it by performing a back and forth movement relative to this position.

This type of periodic movement is called oscillation or movement Oscillatory.

The oscillations of a mass connected to a spring, the movement of a pendulum or the vibrations of a stringed instrument are examples of oscillatory movements.

Chapter I

General information on vibrations & Equation of motion

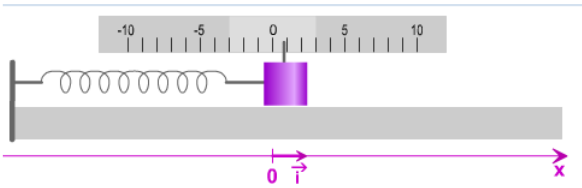
I.1 Introduction

This chapter presents a global description of periodic and harmonic oscillatory movements as well as the various formalisms allowing the determination of the differential equations of motion of conservative systems.

I.2 Definition of a vibration (Oscillation)

A movement of a mechanical system or physical particle around its equilibrium position, which repeats at regular time intervals.

Example: mechanical oscillation (translational system).



The x position of the attached mass to the spring varies around a value 0

(Balance position)

I.3. Périodique mouvement

A periodic movement over time is a movement that repeats itself identically for equal periods of time.

Example: pendulum.

Period: This is the time required for a periodic oscillation to completely occur. The period is therefore expressed in second(s).

Frequency: This is the number of oscillations in a second; it is expressed in s^{-1}

1 oscillation → takes place in T seconds.

f oscillations → take place in 1 second

The pulsation: is defined by the number of revolutions per second (noted ω and measured in rad/s).

$$\omega = 2\pi f = \frac{2\pi}{T}$$

I.4. Harmonic movement

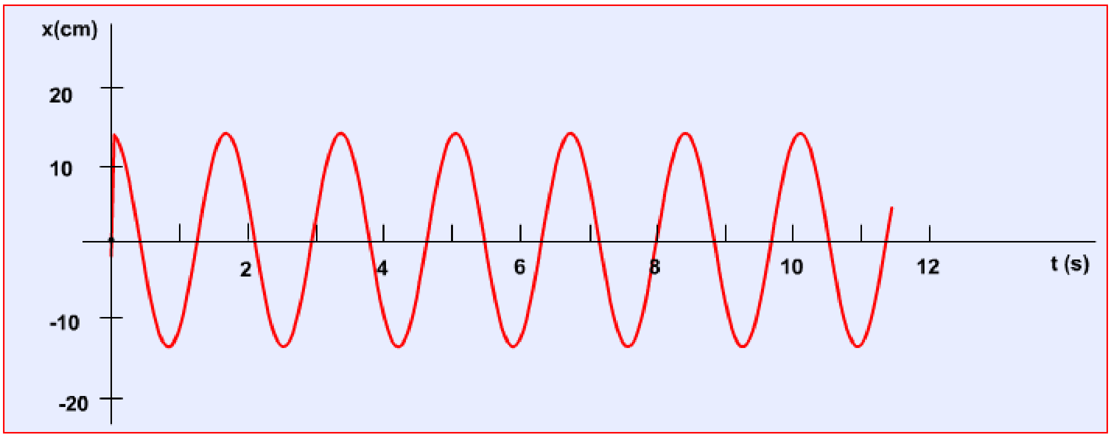
The movement which repeats itself in equal times is called periodic movement. The simplest periodic movement is harmonic movement where the amplitude remains constant (dissipative forces are negligible).

Harmonic motion can be represented mathematically by sine or cosine

Se forme: $x(t) = A\sin(\omega t + \varphi)$ ou bien $x(t) = A\cos(\omega t + \varphi)$

A: the amplitude**, ω:** the pulsation**, φ:** the initial phase**.**

To illustrate this idea we schematize the movement of a mass suspended from a spring



To facilitate calculations, we transform the sinusoidal quantities into exponentials which are simpler to handle. We can consider the oxy plane as a complex plane. The point with coordinates (x, y) corresponds to a complex number z

$z = x + iy$

with $x = r\cos\theta$ and $y = r\sin\theta$

$\cos\theta + i\sin\theta = e^{i\theta}$ with $i^2 = -1$

So :

$z = r(\cos\theta + I\sin\theta) = r\, e^{i\theta}$

with $i^2 = -1$

and from this relationship we can even deduce that:

$\sin\theta = e^{i\theta} - e^{-i\theta}/2$

and $\cos\theta = e^{i\theta} + e^{-i\theta}/2$

1.5Velocity and acceleration in simple harmonic motion

The values of the velocity and acceleration in simple harmonic motion for

$x(t) = A\sin(\omega t + \varphi)$

are given by $dx/dt = \dot{x} = A\omega\cos(wt + \varphi)$

and $d^2X/dt^2 = \ddot{x} = -A\omega^2\sin(wt + \varphi)$

The maximum value of the velocity $A\omega$ is called the velocity amplitude and the acceleration amplitude is given by $A\omega^2$.

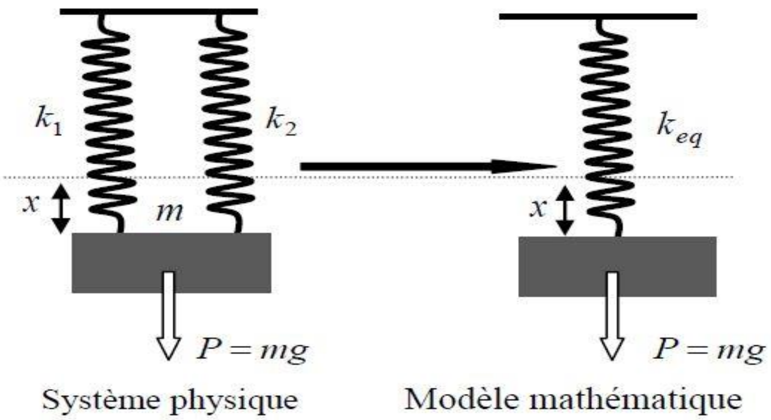
1.6 Mathematical system

The simplification of the complicated vibration system to a simple model representing the real case is carried out by determining the equivalent spring of all the existing springs as well as the equivalent mass of all the masses which constitute the system. In what follows we will give simple examples to understand the equivalent spring and the equivalent mass.

1.6.1 Equivalent spring

In practice we find springs in series and others in parallel.

1stCase : Springs in parallel:



There are two springs k1 and k2; have the same empty length l0 and undergo the same elongation x. When we hang the same mass at the end of the two springs. The equivalent spring of stiffness k has the same elongation.

The equations of the equilibrium system are written as follows:

For the real system:

$$\begin{cases} mg = xk_2 + xk_2 \\ mg = (k_1 + k_2)\, x \quad \Rightarrow k_{eq} = k_1 + k_2 \end{cases}$$

If the system consists of several springs in series, then the constant of the stiffness of the equivalent spring is given by:

$$k_{eq} = \sum_i k_i$$

2emCas: Springs in series

There are two springs k1 and k2; their elongation length x_1 and x_2 respectively; The equivalent spring of stiffness k_{eq} has the elongation $(x = x_1 + x_2)$

$$\begin{cases} k_2x_2 = k_2x_2 \\ mg = k_2x_2 \\ mg = k_{eq}(\, x_1 + x_2) \end{cases} \Rightarrow \begin{cases} x_1 = \dfrac{k_2}{k_1}x_2 \\ \\ k_2x_2 = k_{eq}(\, x_1 + x_2) \end{cases} \Rightarrow k_2x_2 = k_{eq}(\frac{k_2}{k_1}\, x_2 + x_2)$$

$$k_{eq} = \frac{k_1k_2}{k_1 + k_2} \Rightarrow \frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

In the general case where the system is made up of several springs in parallel, the constant of the equivalent spring stiffness can be given as follows:

$$\frac{1}{k_{\mathrm{eq}}} = \sum_i \frac{1}{k_i}$$

1.6.2 Equivalent mass

from the total kinetic energy of the mechanical system; we can find the equivalent mass and the equivalent moment of the system as follows

$$\begin{cases} T_{toutale}(system) = \dfrac{1}{2}(equivqlent\ masse)V^2 \\ \\ T_{toutale}(system) = \dfrac{1}{2}(equivqlent\ moment)\dot{\theta}^2 \end{cases}$$

v linear speed

$\dot{\theta}^2$ Angular speed

Example

use the following system, find the equivalent mass and the equivalent moment.