# Chapter 4: Electrical Transformers

## 1. Transformer :

A transformer is an electro-technical device that transfers electrical energy from one winding to another through magnetic coupling. It is a completely static electrical machine that, in AC operation, allows the modification of certain quantities (voltage, current) without changing their frequency (Figure 1) [1].

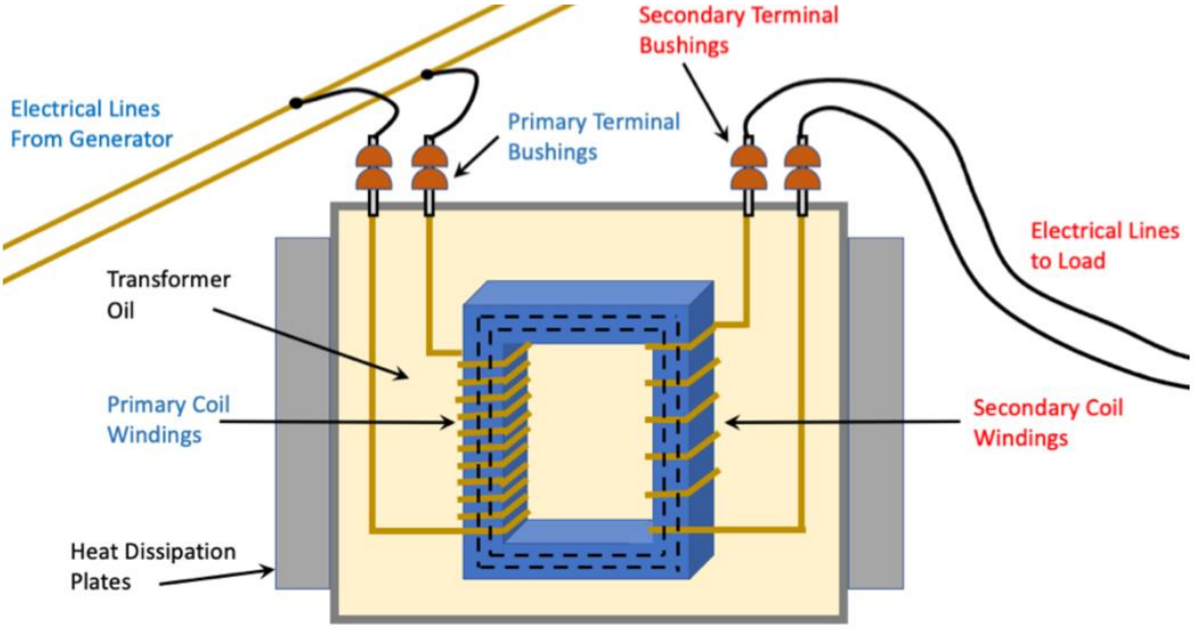
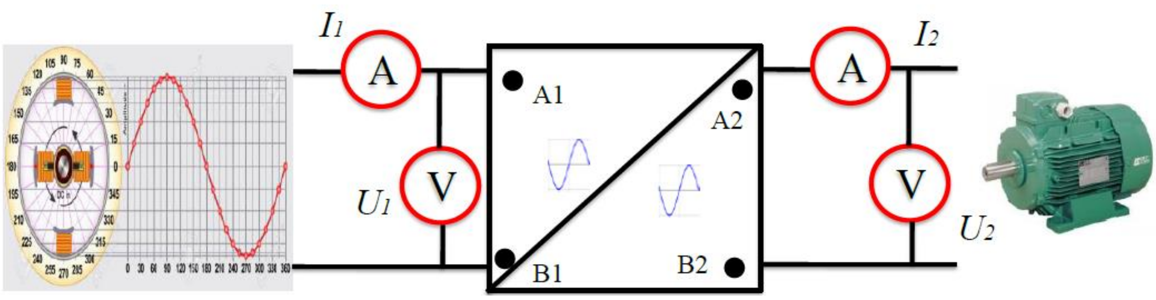



**Figure 1**. Tansformer[1]

From an electrical point of view, it is a quadripole (4 poles or terminals) with:

- A first pair of terminals ($A_1$, $B_1$), forming the input. A generator applies a sinusoidal voltage with an RMS value $\mathbf{U_1}$ (for example, 220 V) across these terminals.
- A second pair of terminals ($A_2$, $B_2$), forming the output. A load connected across these terminals is subjected to a sinusoidal voltage with an RMS value $\mathbf{U_2}$.

Figure 2 illustrates the typical transformer setup[2].




Generator Transformer Receiver

**Figure 2.** Transformer connections.

## 2. Symbolic representation

The three vertical bars in Figure 3 symbolize the magnetic core, which allows the magnetic energy to pass from the primary winding to the secondary winding.

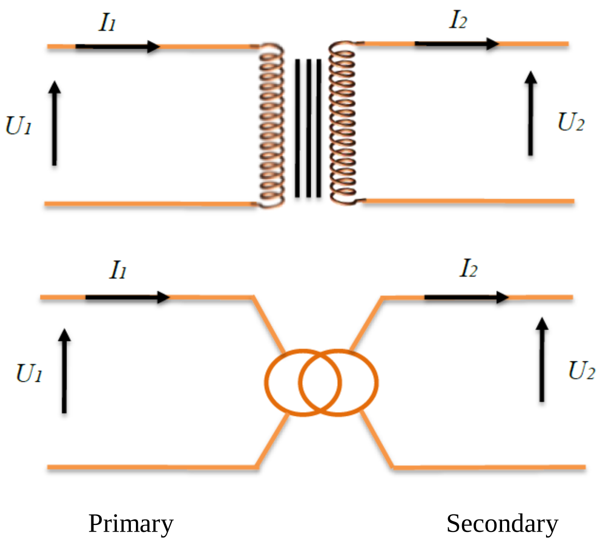



**Figure 3.** Symbolic representation.

The advantage of the transformer is that it supplies the desired voltage $\mathbf{U_2}$ (for example, 24 V) from the available voltage $\mathbf{U_1}$, such as the one delivered by the distribution network.
The transformer receives power at its input terminals and returns almost all of this power to the load connected to its output terminals; in other words, its efficiency is excellent.

If $\mathbf{I_1}$ and $\mathbf{I_2}$ are the RMS values of the sinusoidal currents flowing respectively:

On one hand, between A1 and B1
On the other hand, between A2 and B2

We have: $U_1 I_1 = U_2 I_2$

**1° If $U_2 > U_1 \rightarrow I_2 < I_1$, the transformer is then called:**

- **Step-up transformer**
- **Current step-down transformer**

$U_2$: High Voltage (HV), $U_1$: Low Voltage (LV)

**2° If $U_2 < U_1 \rightarrow I_2 > I_1$, the transformer is then:**

- **Step-down transformer**
- **Current step-up transformer**

$U_1$: High Voltage (HV), $U_2$: Low Voltage (LV)

## 3. Single-phase transformer

The input voltage **$U_1$** is called the primary voltage. The output voltage **$U_2$** is called the secondary voltage.
Let **$r_1$, $r_2$, $l_1$, $l_2$** denote the resistances and leakage inductances of the primary and secondary windings.
**$N_1$** and **$N_2$** represent the number of turns; **$\Phi c$** is the flux common to all the turns, and **$\mathfrak{R}$** is the reluctance of the magnetic circuit.

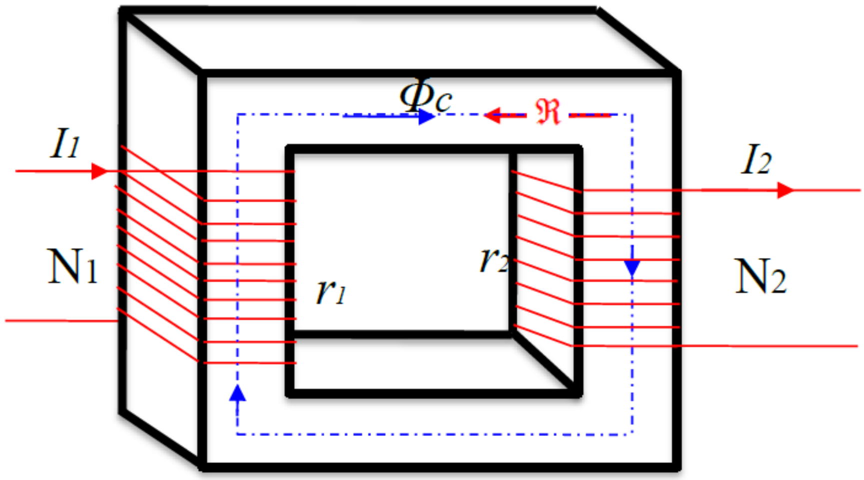



**Figure 4.** Single-phase transformer.

By applying Ohm's law and Kirchhoff's loop law, the operating equations of the transformer can be written as follows:

$$U_1 = r_1 I_1 + l_1 \frac{dI_1}{dt} + N_1 \frac{d\Phi_c}{dt}$$

$$U_2 = -r_2 I_2 - l_2 \frac{dI_2}{dt} - N_2 \frac{d\Phi_c}{dt}$$

The equivalent electrical circuit :

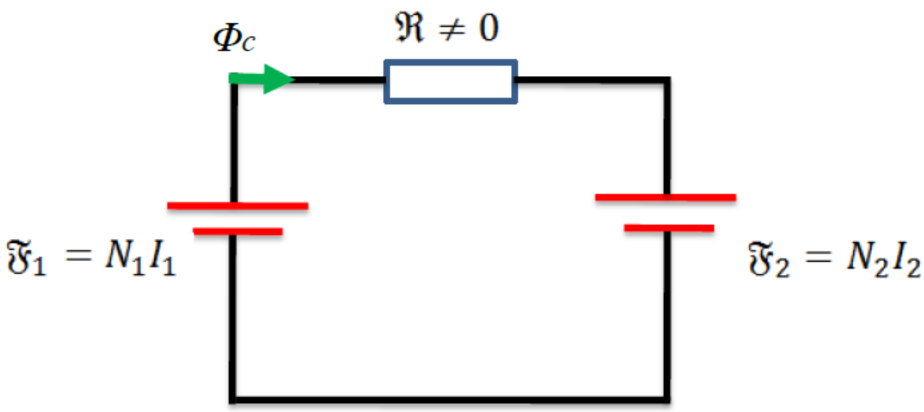



**Figure 5.** Equivalent circuit.

$$\mathfrak{F}_1 - \mathfrak{F}_2 = \Phi_c \mathfrak{R} \leftrightarrow N_1 I_1 - N_2 I_2 = \Phi_c \mathfrak{R}$$

### 3.1. Ideal single-phase transformer

We define the ideal transformer as a transformer with the following characteristics:

The resistance of the wires (in the primary and secondary) is zero (**$r_1 = r_2 = 0$**).
No Joule losses.

The magnetic core is perfect (**$\mu_r = \infty$, $\mathfrak{R} = 0$**).
No magnetic leakage (**$l_1 = l_2 = 0$**).

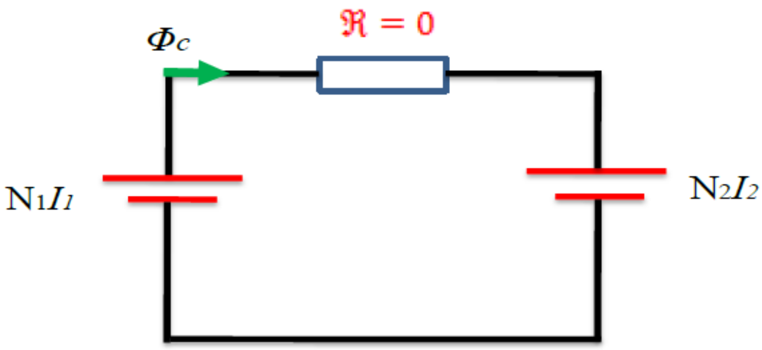



**Figure 6.** Ideal transformer equivalent circuit.

In this case, since the magnetic circuit perfectly channels the field lines, the operating equations become:

$$\begin{cases} U_1 = N_1 \dfrac{d\Phi}{dt} \\ U_2 = N_2 \dfrac{d\Phi}{dt} \\ N_1 I_1 - N_2 I_2 = 0 \end{cases}$$

The transformation ratio **a** is defined as the ratio of the number of turns of the transformer. Thus:

$$a = \frac{N_2}{N_1}$$

Figure 8 shows the symbol of the ideal transformer. In this model, the primary and secondary windings no longer appear.

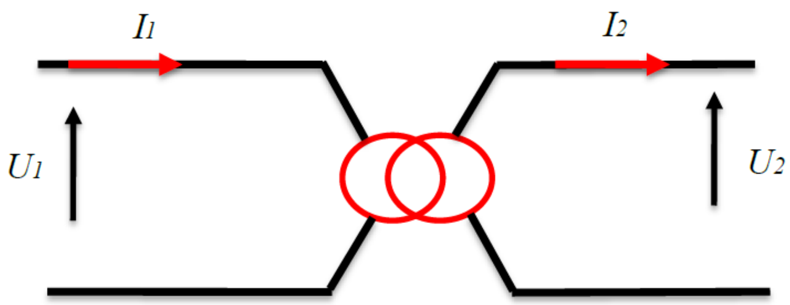



**Figure 8.** Symbol of the ideal transformer.

### Boucherot's formula

Using the equations of the ideal transformer:

$$\begin{cases} U_1 = N_1 \frac{d\Phi}{dt} \\ U_2 = N_2 \frac{d\Phi}{dt} \\ N_1 I_1 - N_2 I_2 = 0 \end{cases}$$

The flux is written as:

$$\Phi = \hat{\Phi} \sin(\omega t)$$

Using the previous equations, we find :

$$U_1 = N_1 \frac{d(\hat{\Phi} \sin(\omega t))}{dt}$$

$$U_1 = N_1 \hat{\Phi} \omega \cos(\omega t)$$

It can be written in the general form:

$$U_1 = U_{ef1} \sqrt{2} \cos(\omega t)$$

$$\hat{\Phi} = B.S \qquad ; \omega = 2\pi f$$

$$\boldsymbol{U_1 = \frac{2\pi}{\sqrt{2}} . N_1 . S . f . B}$$

## 3.2. Real single-phase transformer

A real transformer has winding resistances ($r_1 = r_2 \neq 0$) that are not negligible. Therefore, the transformer will experience Joule losses (also called copper losses):

$$L_{joule} = L_{copper} = r_1 I_1{}^2 + r_2 I_2{}^2$$

Part of the primary current produces flux not linked to the secondary, called the primary leakage flux $\Phi_{11}$.
Similarly, part of the secondary current produces flux not linked to the primary, called the secondary leakage flux $\Phi_{22}$.
The primary flux is:

$$\Phi_1 = \Phi_m + \Phi_{11} = \Phi_m + \frac{I_1 L_1}{n_1}$$

The secondary flux is:

$$\Phi_2 = \Phi_m + \Phi_{22} = \Phi_m + \frac{I_2 L_2}{n_2}$$

These losses, called magnetic losses, are proportional to the current in the corresponding winding and are quantified by the primary and secondary inductances, $L_1$ and $L_2$. These losses are of an inductive nature.
The core of a real transformer is not perfect, its relative permeability $\mu_r$ is finite, and the magnetizing current is not negligible. Indeed, the core is represented by a magnetizing reactance $X_m$.
Iron losses are mainly caused by hysteresis and eddy currents. These losses are represented by a resistance $R_c$ connected in parallel with $X_m$.
The following equations describe the operation of a real transformer:

$$U_1 = r_1 I_1 + l_1 \frac{dI_1}{dt} + N_1 \frac{d\Phi_c}{dt}$$

$$U_2 = -r_2 I_2 - l_2 \frac{dI_2}{dt} - N_2 \frac{d\Phi_c}{dt}$$

$$N_1 I_1 - N_2 I_2 = \Phi_c \Re$$

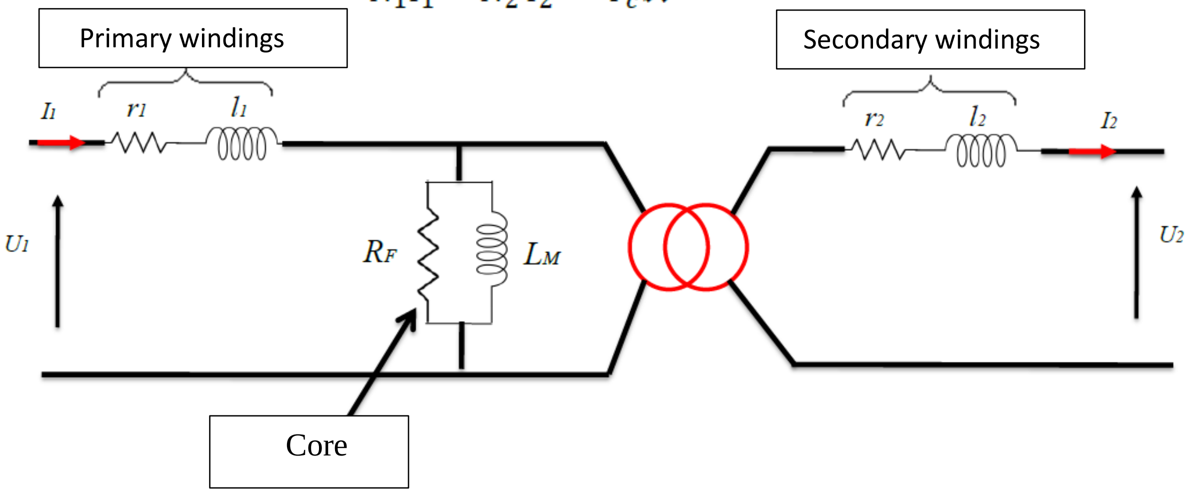



**Figure 9.** Real transformer equivalent circuit.

The consequences of these parasitic phenomena are:

- The transformer efficiency is less than 100%.
- The voltage ratio between primary and secondary will not be exactly equal to the turn's ratio.
- The secondary voltage will change with the load.

Indeed, the secondary voltage $U_2$ under load is different from the no-load secondary voltage $U_{20}$.
The secondary voltage drop under load is defined as the difference between the RMS values of these voltages:

$$\Delta U_2 = U_{2V} - U_2$$

## 4. Transformer efficiency

The power delivered by the transformer is the useful power $P_u$, while the power it absorbs is the active power $P_a$.
The transformer efficiency is defined as the ratio of $P_u$ to $P_a$

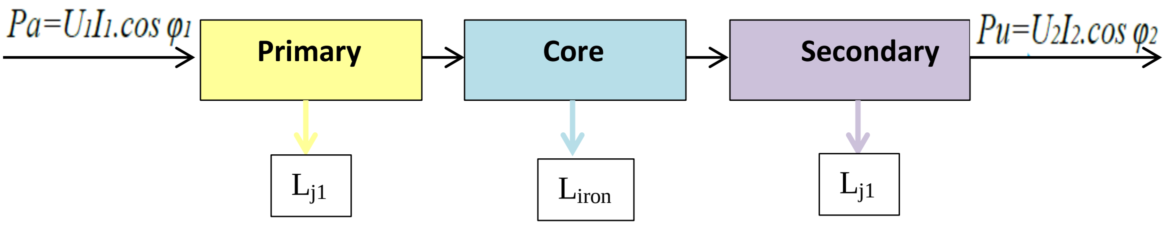



**Figure 10**. Efficiency balance.

$$P_a = P_u + L_{j1} + L_{j2} + L_{iron}$$

$$= P_u + L_{copper} + L_{iron}$$

$$P_a = P_u + L_{copper} + L_{iron}$$

$$\eta = \frac{P_u}{P_u + L_{copper} + L_{iron}}$$ ; $\eta < 1$, but close to 1.

## 5. Determination of the parameters:

### 5.1. Iron losses determination

To determine the iron losses, an open-circuit test is performed (no-load test) at the rated voltage on the primary. The equivalent circuit is then referred to the primary.

The figure below shows the equivalent circuit in open-circuit condition:

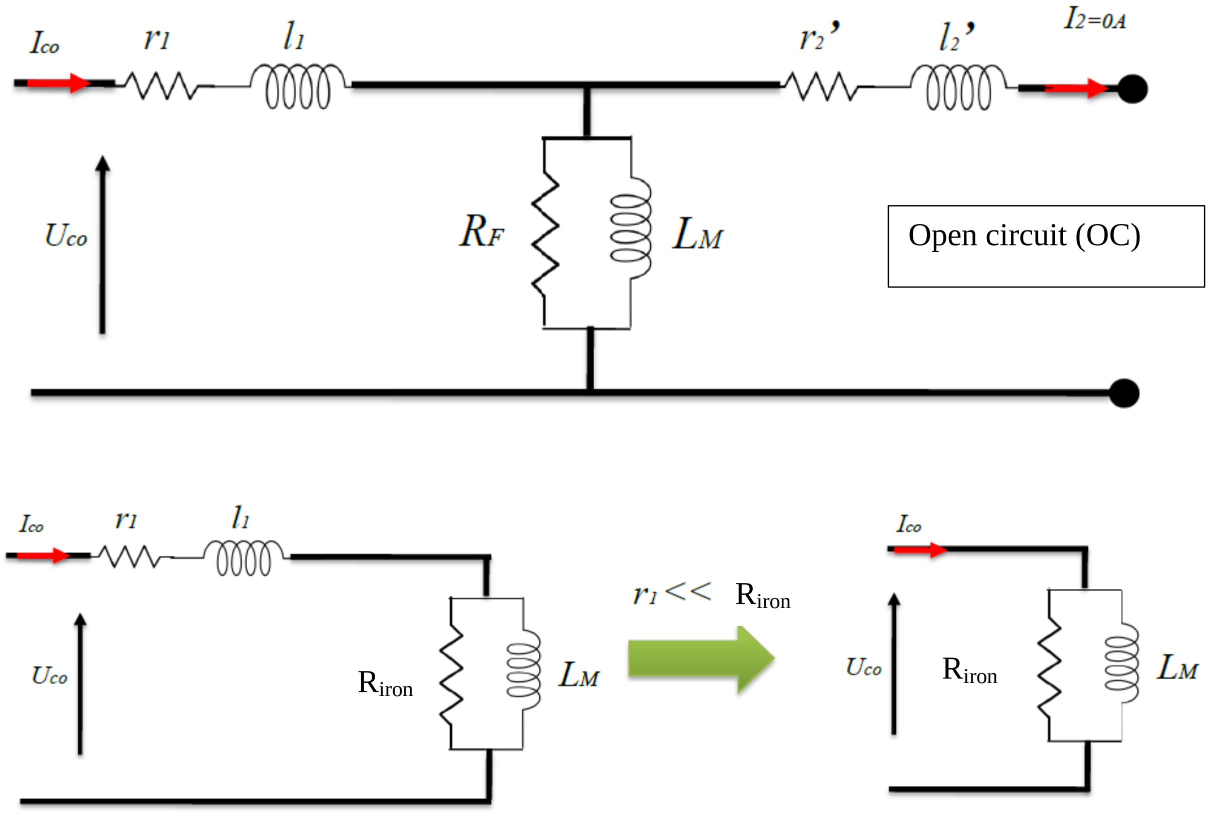



**Figure 11.** Open-circuit equivalent circuit .

Open-circuit tests are used to determine the magnetizing branch parameters.

We measure the parameters: $P_{CO}$ ,$V_{CO}$, $I_{CO}$

With the secondary open: $I_2 = 0$ A $\longrightarrow L_{j2} = 0$, $P_u = 0$

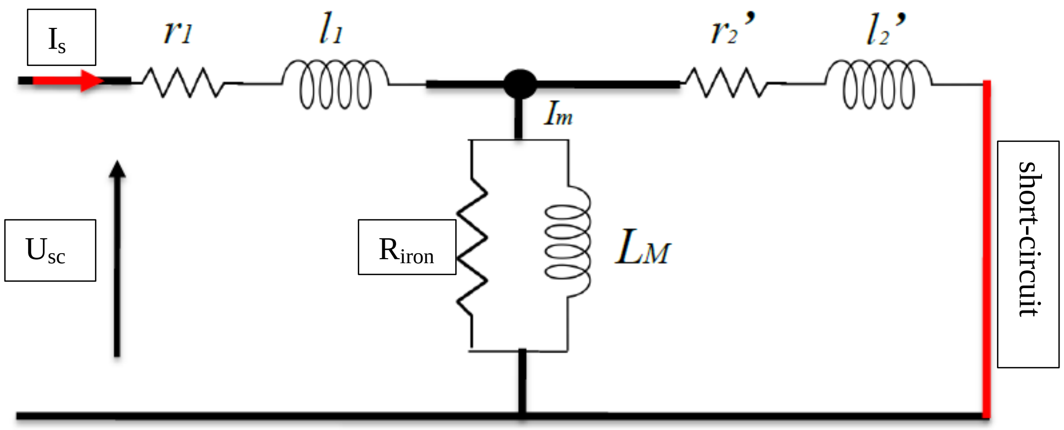
The wattmeter measures $P_{CO}$:

$$\begin{aligned} P_{co} &= P_{\mathrm{u}} + L_{\mathrm{j1}} + L_{\mathrm{j2}} + L_{iron} \\ &= 0 + L_{\mathrm{j1}} + 0 + L_{iron} \end{aligned}$$

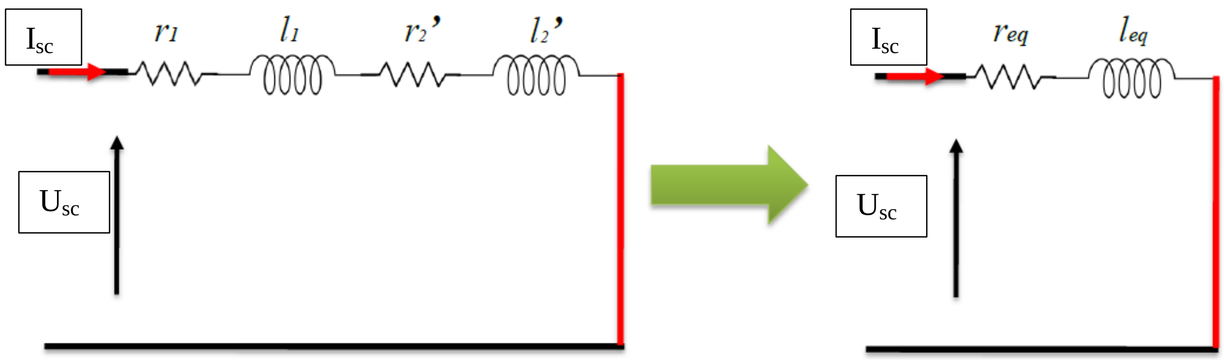
Since $L_{\mathrm{j1}} \ll L_{iron}$, we have $P_{co} \approx L_{iron}$ .

## 5.2. Determination of Joule (copper) losses

To determine the copper losses, a short-circuit test is performed at reduced voltage and rated secondary current.




$R_{iron} \gg r_1 \rightarrow I_M = 0A$; The magnetizing branch is negligible. The circuit then becomes:




**Figur 12.** Short-circuit equivalent circuit.

$$P_{sc} = P_u + L_j^1 + L_j^2 + L_{iron}$$
$$= 0 + L_j^1 + L_j^2 + 0 \approx L_j \approx L_{copper}$$

$$U_2 = 0 \rightarrow P_u = 0$$

$$I_M = 0 \rightarrow L_{iron} = 0$$

So : $\boldsymbol{P_{sc} \approx P_j}$

The short-circuit tests allow us to determine the parameters of the windings.

## ➢ Application No. 1

Laboratory tests on a 1.5 kVA, 220/110 V, 60 Hz transformer yielded the following measurements:

- **Open-circuit test** (high-voltage side open-circuited):
  $V_{oc}$ = 110 V, $I_{oc}$ = 0.4 A, $P_{oc}$ = 25 W
- **Short-circuit test** (high-voltage side short-circuited):
  $V_{sc}$ = 8.25 V, $I_{sc}$ = 13.6 A, $P_{sc}$ = 40 W
- **DC measurement of the winding resistances**:
  $R_{hv}$ = 0.413 Ω, $R_{lv}$ = 0.113 Ω

**Questions:**

1. If the maximum flux in the transformer core is to be 2 mWb, what should be the number of turns of the primary and secondary windings?
2. Determine the various transformer parameters from the tests.
3. Give the equivalent circuit referred to the primary and then to the secondary of this transformer.
4. If the transformer delivers 110 V at its rated power to an inductive load with a power factor of 0.8, what is the corresponding primary voltage?
5. What would be the efficiency of this transformer under these conditions?

[1] https://mepacademy.com/how-electrical-transformers-work.

[2] EFCA