

**Exercise 1.**

The FIRST and FOLLOW set for non-terminals A and B in the following productions:

**P:**

$$A \rightarrow BAa \mid \varepsilon$$

$$B \rightarrow bBc \mid AA$$

	First	Follow
A	$\varepsilon$ a b	# a b c
B	$\varepsilon$ a b	a b c

**Exercise 2.**

We define the following grammar  $G = \langle N, T, P, S \rangle$ :

$$P: \quad S \rightarrow A a A b \mid B b B a$$

$$A \rightarrow \varepsilon$$

$$B \rightarrow \varepsilon$$

a) Construction of the LL(1) parsing table for G.

- The FIRST and FOLLOW sets

	First	Follow
S	a b	#
A	$\varepsilon$	a b
B	$\varepsilon$	a b

- The LL(1) parsing table for G.

	a	b	#
S	$S \rightarrow A a A b$	$S \rightarrow B b B a$	
A	$A \rightarrow \varepsilon$	$A \rightarrow \varepsilon$	
B	$B \rightarrow \varepsilon$	$B \rightarrow \varepsilon$	

- This grammar is LL(1) because the LL(1) parsing table is unambiguous (single-defined).

**b) Analysis of the string: abb**

Stack	Remaining to be analyzed	Action
#S	abb#	Pop (S), stack (bAaA)
#bAaA	abb#	Pop (A)
#bAa	abb#	Pop (a), Move Forward()
#bA	bb#	Pop (A)
#b	bb#	Pop (b), Move Forward()
#	b#	" error "

**Analysis of the string: ab**

Stack	Remaining to be analyzed	Action
#S	ab#	Pop (S), stack (bAaA)
#bAaA	ab#	Pop (A)
#bAa	ab#	Pop (a), Move Forward()
#bA	b#	Pop (A)
#b	b#	Pop (b), Move Forward()
#	#	"String accepted"

**Exercise 3.**

1. The grammar is **not LL(1)** because it is left-recursive.

**2. Elimination of left recursion:**

The left-recursive production rules  $T \rightarrow T, S \mid S$  are replaced by:  $T \rightarrow ST'$

$T' \rightarrow, ST' \mid \epsilon$

The grammar becomes:  $S \rightarrow a \mid b \mid (T)$

$T \rightarrow ST'$

$T' \rightarrow, ST' \mid \epsilon$

**Grammar factorization:** The resulting grammar is already **factored**.

**3. Proof that the grammar obtained in step 2 is LL(1):**

Compute the **FIRST** and **FOLLOW** sets of the grammar  $G$ :

Non-terminal symbol	First	Follow
S	a b (	# , )
T	a b (	)
T'	, ε	)

**Proof:**

For every pair of rules  $A \rightarrow a \mid bA$  and  $B \rightarrow a \mid b$ , we have:

$$\text{FIRST}(a.\text{FOLLOW}(A)) \cap \text{FIRST}(b.\text{FOLLOW}(A)) = \emptyset$$

- For the rules  $S \rightarrow a \mid b$ :  
 $\text{FIRST}(a.\text{FOLLOW}(S)) \cap \text{FIRST}(b.\text{FOLLOW}(S)) = \emptyset$
- For the rules  $S \rightarrow a \mid (T)$ :  
 $\text{FIRST}(a.\text{FOLLOW}(S)) \cap \text{FIRST}((T).\text{FOLLOW}(S)) = \emptyset$
- For the rules  $S \rightarrow b \mid (T)$ :  
 $\text{FIRST}(b.\text{FOLLOW}(S)) \cap \text{FIRST}((T).\text{FOLLOW}(S)) = \emptyset$
- For the rules  $T' \rightarrow ,ST' \mid \epsilon$ :  
 $\text{FIRST}(,ST'.\text{FOLLOW}(S)) \cap \text{FIRST}(\epsilon.\text{FOLLOW}(T')) = \emptyset$

Therefore, the grammar obtained in step 2 is LL(1).

The grammar analysis table.

	has	b	(	)	,	#
S	$S \rightarrow a$	$S \rightarrow b$	$ST \rightarrow )$			
T	$T \rightarrow ST'$	$T \rightarrow ST'$	$T \rightarrow ST'$			
T'				$T' \rightarrow \epsilon$	$T' \rightarrow ,ST'$	

4. The analysis of the string (a,(b,a),a) :

Stack Contents	Remaining to be analyzed	Action
#S	(a,(b,a),a)#	Replace S with )T(

#)T(	(a,(b,a),a)#	Move forward
#)T	a,(b,a),a)#	Replace T with T <sup>s</sup>
#) T ' S	a,(b,a),a)#	Replace S with a
#) T ' a	a,(b,a),a)#	Move forward
#) T ' '	,(b,a),a)#	Replace T ' with T ' S,
#) T ' T ' S,	,(b,a),a)#	Move forward
#) T ' T ' S	(b,a),a)#	Replace S with )T(
#) T ' T ' )T(	(b,a),a)#	Move forward
#) T ' T ' )T	b,a),a)#	Replace T with T <sup>s</sup>
#) T ' T ' ) T ' S	b,a),a)#	Replace S with b
#) T ' T ' ) T ' b	b,a),a)#	Move forward
#) T ' T ' ) T ' '	,a),a)#	Replace T ' with T ' S,
#) T ' T ' ) T ' S,	,a),a)#	Move forward
#) T ' T ' ) T ' S	a),a)#	Replace S with a
#) T ' T ' ) T ' a	a),a)#	Move forward
#) T ' T ' ) T ' '	),has)#	Pop (T ')
#) T ' T ' )	),has)#	Move forward
#) T ' T ' '	,has)#	Replace T ' with T ' S,
#) T ' T ' S,	,has)#	Move forward
#) T ' T ' S	has)#	Replace S with a
#) T ' T ' a	has)#	Move forward
#) T ' T ' '	)#	Pop (T ')
#) T ' '	)#	Pop (T ')
#)	#)	Move forward

#	#	“String accepted”
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**Exercise 4.**

We define the following grammar  $G = \langle N, T, P, S \rangle$ :

$P : S \rightarrow (A) \mid a$

$A \rightarrow SB$

$B \rightarrow ,S \mid S a$

**1. Verify whether the grammar obtained is LL(1) or not:**

- Compute the **FIRST** and **FOLLOW** sets :

	First	Following
S	( a	a ( , # )
A	( a	)
B	( a ,	)

**- verification :**

For every pair of rules  $A \rightarrow \alpha \mid \beta$  We have:

For:  $S \rightarrow (A) \mid a$

$\text{First} ( (A) . \text{FOLLOW}(S) ) \cap \text{First} ( a . \text{FOLLOW} (S) ) = \Phi$

For:  $B \rightarrow ,S \mid S a$

$\text{First} ( ,S . \text{FOLLOW} (B) ) \cap \text{First} ( S a . \text{FOLLOW} (B) ) = \Phi$

Therefore, the grammar is **LL(1)**

**2. Writing the syntax analysis algorithm** that recognizes the words of the language  $L(G)L(G)L(G)$  using the **recursive descent method**.

Add the rule  $Z \rightarrow S\#$

```

Procedure Z()
Begin
  S();
  if tc = '#' then
    "string accepted" ;
  else "Error" ;
  end if
End

```

```

Procedure A()
Begin
  S();
  B();
End

```

```

Procedure S()
Begin
  if tc = '(' then
    tc ← ts ;
    A();
  if tc = ')' then
    tc ← ts ;
  else "Error"
  end if
else
  if tc = 'a' then
    tc ← ts ;
  else "Error"
  end if
end if
end

```

```

Procedure B()
Begin
  if tc = ',' then
    tc ← ts ;
    S();
  else
    S();
  if tc = 'a' then
    Tc ← ts ;
    if tc = ')' then
      Tc ← ts
    else "Erreur"
    end if
  else "Error"
  end if
end if
end

```

**3. Analysis of the string: (( a,a))#**

	Remaining to be analyzed	Action
Z	((a,a))#	Call(S)
ZS	((a,a))#	Move forward (), Call(A)
ZSA	(a,a)#	Call(S)
ZSAS	(a,a)#	Move forward (), Call(A)
ZSASA	a,a)#	Call(S)
ZSASAS	a,a)#	Move forward(),Return(S)
ZSASA	,a)#	Call(B)
ZSASAB	,a)#	Move forward (), Call(S)
ZSASABS	a)#	Move forward (),Return(S)
ZSASAB	)#	Return (B)
ZSASA	)#	Return (A)
ZSAS	)#	Move forward (),Return (S)
ZSA	)#	Call(B)
ZSAB	)#	Call(s)
ZSABS	)#	" Error "

**3+. Analysis of the string: (a,a)#**

	Remaining to be analyzed	Action
Z	(a,a)#	Call(s)
ZS	(a,a)#	Move forward (), Call(A)
ZSA	a,a)#	Call(S)
ZSAS	a,a)#	Move forward (), Return(S)

ZSA	,a)#	Call(B)
ZSAB	,a)#	Move forward(),Call(S)
ZSABS	a)#	Move forward (), Return(S)
ZSAB	)#	Return(B)
ZSA	)#	Return(A)
ZS	)#	Move forward (), Return(S)
Z	#	"Accepted string"