Series of Exercises 05

Exercise 1

Returning to the definition, study whether the following generalized integrals have meaning or not and give their possible values.

❶ $I_n = \int_0^{+\infty} x^n \exp(-x)dx,\ n \in \mathbb{N}.$

❷ $J_n = \int_0^1 \ln^n x dx,\ n \in \mathbb{N}.$

❸ $I = \int_0^2 \frac{dx}{\sqrt{(2x - x^2)}}.$

Exercise 2

Study the convergence of the following generalized integrals:

❶ $I_1 = \int_0^{\frac{1}{2}} \frac{\sin x}{x^\alpha \left|\ln x\right|^\beta} dx.$

❷ $I_2 = \int_0^{\frac{1}{2}} \frac{\ln(1 + x\sin x)}{\left|x\ln x\right|^\delta} dx.$

❸ $I_3 = \int_0^{\frac{1}{2}} \frac{\arctan(x^2)}{x^3 \left|\ln x\right|} dx.$

❹ $I_4 = \int_1^2 \frac{\left|x - 1\right|^\alpha}{\left|\ln x\right|^\beta} dx.$

❺ $I_5 = \int_1^2 \frac{dx}{\sqrt{(x-1)(2-x)}}.$

Exercise 3

Study the absolute convergence and semi-convergence of the following generalized integrals:

❶ $I_1 = \int_1^{+\infty} \frac{\sin x}{x^\theta} dx,\ \theta \in \mathbb{R}_+^*.$

❷ $I_2 = \int_1^{+\infty} \frac{\cos x}{\sqrt{x} + \cos x} dx.$

Exercise 4 (Personal work)

Let α be a given positive real.

❶ Show that $F(\alpha) \int_0^{+\infty} \frac{\cos(\alpha x)}{1+x^2} dx$, converges absolutely.

❷ Show that $\lim_{\alpha \to +\infty} F(\alpha) = 0$, for all $\alpha > 0$.

Exercise 5 (Personal work)

Let P and Q two polynomials, such that $\deg(P) < \deg(Q)$. Study the convergence of the following generalized integral:

$$I = \int_a^{+\infty} \frac{P(x)}{Q(x)} \cos(x) dx, \; Q \text{ does not vanish on } [a, +\infty[\,.$$