Exercise Set: Tests on a Mean and a Proportion

Known variance, unknown variance, proportions, two-sided tests

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EXERCISE STATEMENTS

Exercise 1: Test on a Mean (known variance, two-sided)

A manufacturer claims that the mean weight of its flour packages is $\mu_0 = 500$ g. Previous studies have shown that the population standard deviation is known to be $\sigma = 8$ g.

A random sample of $n = 36$ packages yields a sample mean of $\bar{X} = 503$ g.

We want to test, at the $\alpha = 5\%$ significance level:

$$H_0 : \mu = 500 \quad \text{versus} \quad H_1 : \mu \neq 500$$

Questions:

1. Explain why a Z-test is appropriate and write the test statistic.
2. Compute the observed value of the statistic.
3. Determine the critical region at the 5% significance level.
4. State your conclusion.

Exercise 2: Test on a Mean (unknown variance, two-sided)

A company produces metal pieces with an advertised mean length $\mu_0 = 100$ mm.

A sample of $n = 20$ pieces is taken, yielding:

$$\bar{X} = 102 \text{ mm}, \qquad s = 6 \text{ mm}.$$

We want to test, at the $\alpha = 5\%$ level:

$$H_0 : \mu = 100 \quad \text{versus} \quad H_1 : \mu \neq 100$$

Questions:

1. State the distribution of the test statistic under H_0.
2. Compute the test statistic.
3. Determine the critical region at the 5% level.
4. Decide whether to reject H_0 and interpret.

Exercise 3: Test on a Mean (unknown variance, two-sided)

A laboratory monitors the concentration of a chemical substance. The regulatory standard requires a mean concentration of $\mu_0 = 50$ units.

A sample of $n = 16$ observations gives:

$$\bar{X} = 47, \qquad s = 4$$

We test at the $\alpha = 5\%$ significance level:

$$H_0 : \mu = 50 \quad \text{versus} \quad H_1 : \mu \neq 50$$

Questions:

1. Write the test statistic and its distribution under H_0.
2. Calculate the observed value of the statistic.
3. Determine the rejection region at the 5% level.
4. Conclude: what can we say about the mean concentration?

Exercise 4: Test on a Proportion (two-sided)

A company guarantees that at least 40% of its products are conforming, i.e. $p_0 = 0.40$.

During a quality control check, a sample of $n = 150$ products is taken, and $X = 75$ of them are conforming.

Thus the sample proportion is:

$$\hat{p} = \frac{75}{150} = 0.50$$

We test at the significance level $\alpha = 5\%$:

$$H_0 : p = 0.40 \quad \text{versus} \quad H_1 : p \neq 0.40$$

Questions:

1. Explain why a Z-test for proportions is appropriate.
2. Compute the test statistic.
3. Determine the critical region for a two-sided test at 5%.
4. Conclude the test and interpret the result.