Chapter 3:

Classification of Heat Exchangers

1st year Master

Chemical Engineering & Pharmaceutical Engineering

Heat exchangers are devices in which thermal exchange takes place between two fluids at different temperatures without mixing them.

Within a heat exchanger, three modes of heat transfer can be found:

- **Thermal conduction**, which is the propagation of heat from molecule to molecule within a body or through several superimposed and non-reflective bodies.
- **Convection**, which is characterized by the propagation and transport of heat by moving molecules that warm up when in contact with a hot body and carry this thermal energy to a cold body.
- **Radiation**, in which heat transfer occurs through electromagnetic vibrations that propagate in a straight line without any medium; i.e., any body placed in a vacuum emits energy, and any other body located along its path absorbs all or part of this energy. In general, radiation heat transfer in a heat exchanger is negligible.

Thus, the heat transfer that occurs within a heat exchanger involves **convective transfer** in each of the two fluids and **conductive transfer** through the wall that separates them. Generally, radiation is weak and can be neglected.

In general, there are **three classes of heat exchangers**:

1. **Direct transfer exchangers**.
2. **Thermal stokage exchangers**.
3. **Direct contact exchangers.**

2. General Principle

The principle consists of circulating two fluids through conduits that bring them into **thermal contact**. These fluids are in thermal contact through a **metal wall**, which promotes heat transfer. In general, the **hot fluid transfers heat** to the **cold fluid**.

The main challenge lies in defining a **sufficient heat transfer surface** between the two fluids to transfer the required amount of heat for a given configuration. The quantity of heat transferred depends not only on the **exchange surface area** between the two fluids but also on many other parameters.

The **heat fluxes** also depend on:

- the inlet temperatures,
- the thermal properties of the fluids (specific heat capacities, thermal conductivity),
- the convective heat transfer coefficients.

3. Geometric Configurations

3.1 Coaxial (Simple) Tubular Heat Exchangers

One of the fluids flows through the **annular space** between the two tubes, while the other flows through the **inner (central) tube**.

For this configuration, two types of operation are distinguished:

- When the two fluids flow in the **same direction**, the exchanger is called a **parallel-flow** (or **co-current**) heat exchanger.
- When the fluids flow in **opposite directions**, the exchanger is a **counterflow** heat exchanger.

This type of exchanger is commonly used in the **refrigeration industry**, particularly for **water-cooled condensers** or in **chilled-water production units**.

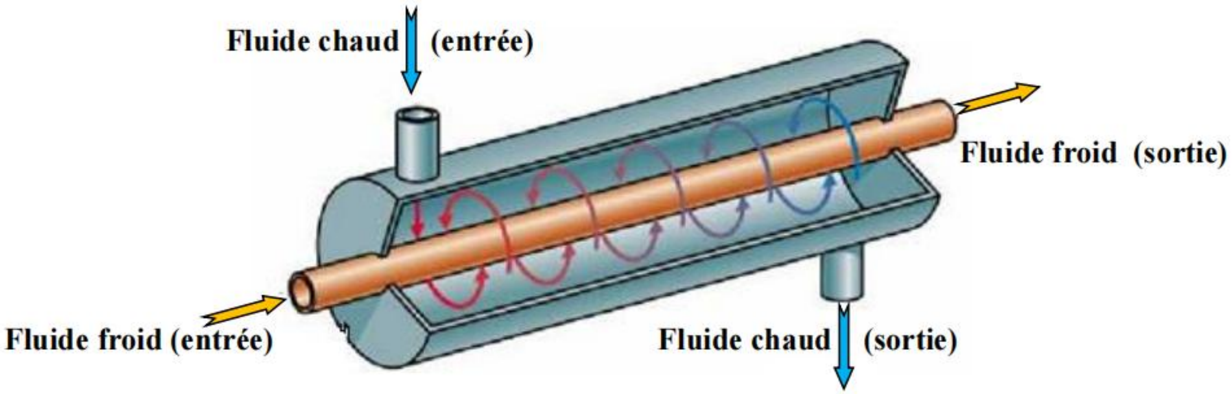


Figure 1 : Échangeur tubulaire simple. [27]

It is the **simplest type of heat exchanger** that can be designed, but with this type it is difficult to achieve large heat transfer surfaces without ending up with very bulky equipment.

Therefore, it is necessary to develop **other exchanger geometries** to overcome this limitation.

3.2 Shell-and-Tube Heat Exchangers (with Complex Tube Bundles)

In this type of heat exchanger, one of the fluids flows through the **shell** around a set of tubes, while the other fluid flows **inside the tubes**.

It is often composed of a **bundle of tubes** running longitudinally through a tank, hence the name **multi-tube heat exchanger**.

The circulation of the fluid through the tubes is **forced by the placement of baffles**, so that it makes **one or several passes** back and forth along the exchanger.

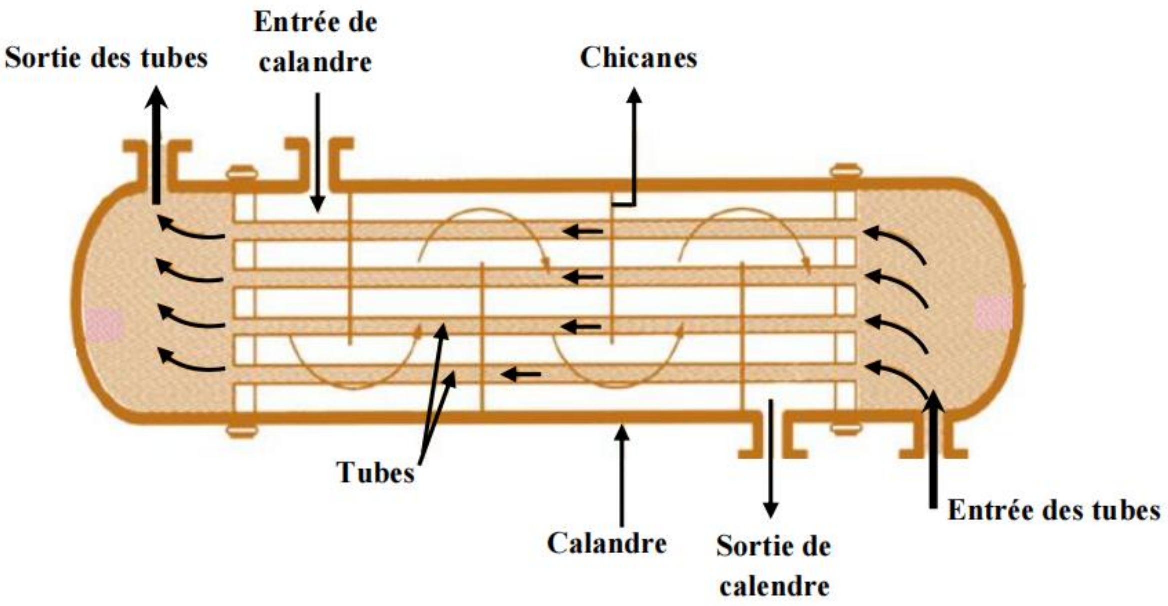


Figure 2 : Principe d'un échangeur de chaleur tubes/calandre. [11]

The principle is illustrated in **Figure 2**. These exchangers consist either of a **single tube (coil)** or of a **bundle of tubes connected in parallel**, enclosed within a shell called the **calandria (or shell)**.

The placement of **baffles inside the shell** increases turbulence and improves heat exchange efficiency. In general, the role of the baffles can be summarized in two main points:

1. To increase the rigidity of the tube bundle in order to prevent vibration phenomena.

2. To increase the velocity of the fluid.

The flow of the hot fluid through **metal tubes** (steel, copper, stainless steel, etc.) minimizes **heat losses**.

In **Figure 3**, a real schematic of a **multitube heat exchanger** manufactured in Spain and used in the **industrial zone of Skikda** is shown.
This exchanger is an **ethylene reheater** (ethylene is a gas used as a monomer in polymerization reactions). Moreover, each industrial unit is equipped with an **identification plate**

From this plate, one can determine its characteristics, such as:

- the nature of the fluid on the tube side and on the shell side,
- the operating pressure of each fluid,
- the operating temperature,
- the hydrostatic test pressure,
- the weight of the tubes, etc.

Figure 3: Schematic of a multitube heat exchanger — *ethylene reheater*, disassembly of the tube side and shell side for retesting. (CP/2K, Sonatrach Skikda)

Three classes of equipment can be distinguished:

- **Fixed tube sheet exchangers:** welded to the shell, they can only be used when the temperature difference between the hot and cold fluids is small enough for the tube bundle expansion to remain acceptable.
- **Floating head exchangers:** one of the tube sheets is fixed, while the second sheet, which has a smaller diameter, supports the return box and can slide freely inside the cover that closes the shell.

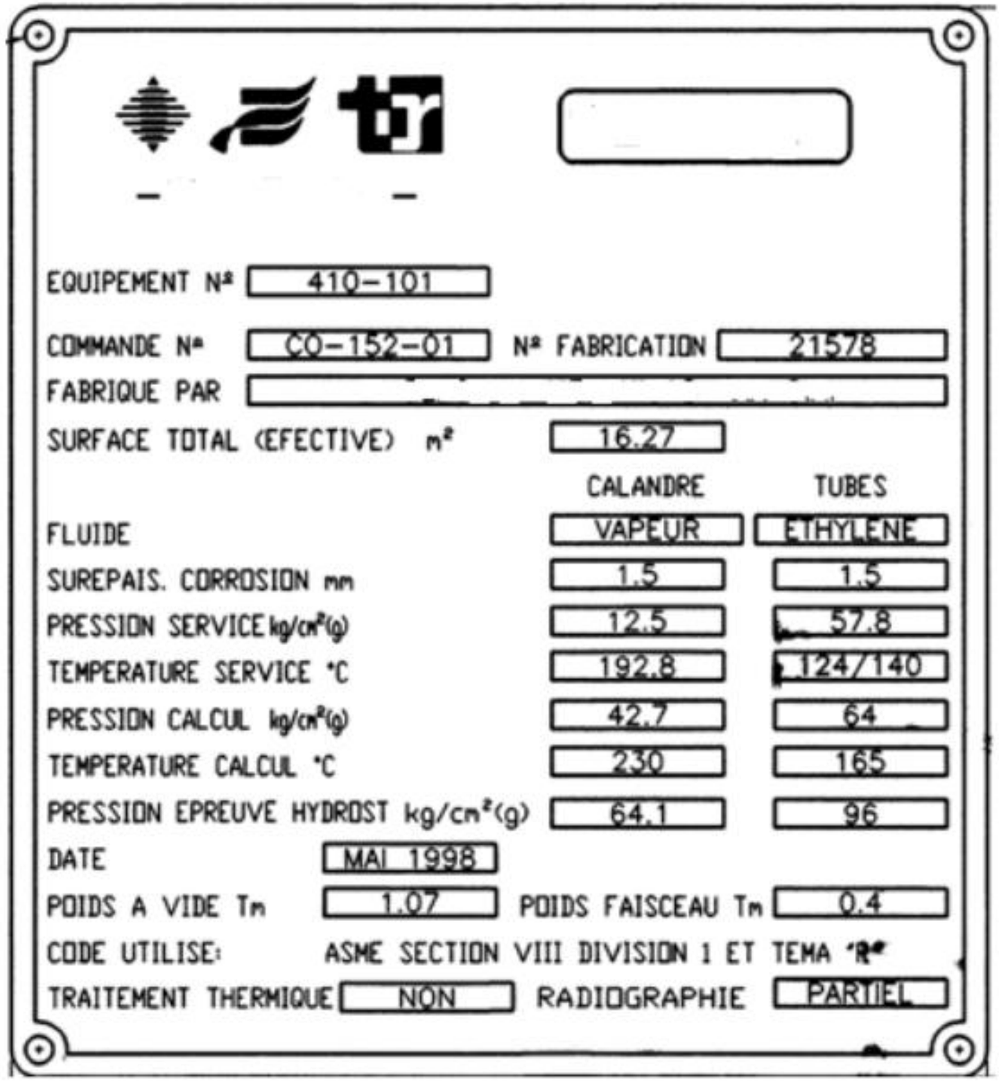


Figure 4: Identification plate attached to the ethylene reheater shown in Figure 3.

These exchangers allow for the thermal expansion of the tube bundle as well as mechanical cleaning, and they make up almost all of the exchangers used in the refinery of the Skikda industrial zone.

U-tube heat exchangers:

In this type, the tube sheet is eliminated by using bent tubes, while maintaining the expansion properties of the floating head design. The cost savings achieved by removing the tube sheet are offset by the impossibility of mechanically cleaning the inside of the tubes. Such tube bundles are mainly used in **steam reboilers**.

3.2.1- **Heat Exchanger 1-2**

This type is the simplest shell-and-tube heat exchanger:

the fluid flowing through the shell makes a single pass, while the fluid on the tube side makes two (or 2n) passes.

For the heat exchanger shown in **Figure (2)**, the fluid makes a single pass both on the shell side and within the tubes.

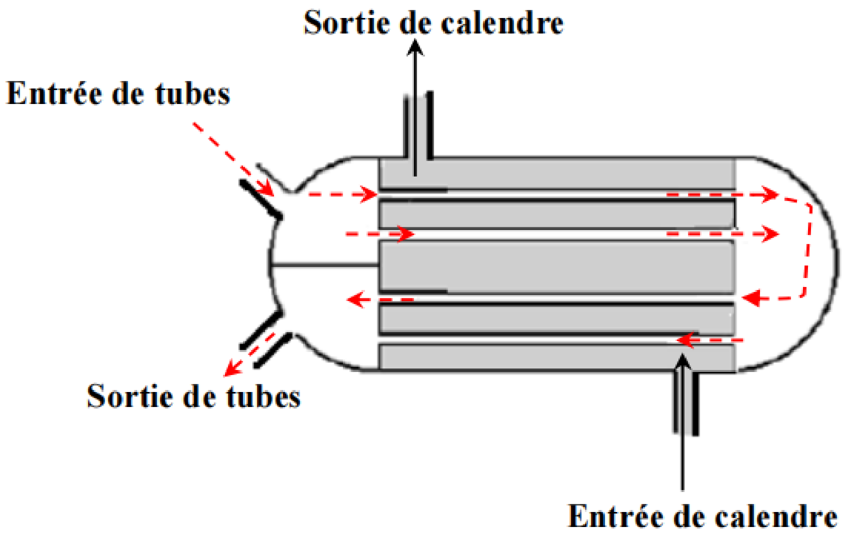


Figure 5: Principle of a 1–2 Heat Exchanger

3.2.2- **Heat Exchanger 2-4**

When a 1–2 exchanger does not provide an efficiency higher than 0.75, it is possible to approach the counter-current configuration by allowing the shell-side fluid to make two (or more) passes.

The 2–4 heat exchanger includes a longitudinal baffle so that the shell-side fluid makes two passes, while the tube-side fluid makes four (or 4n) passes

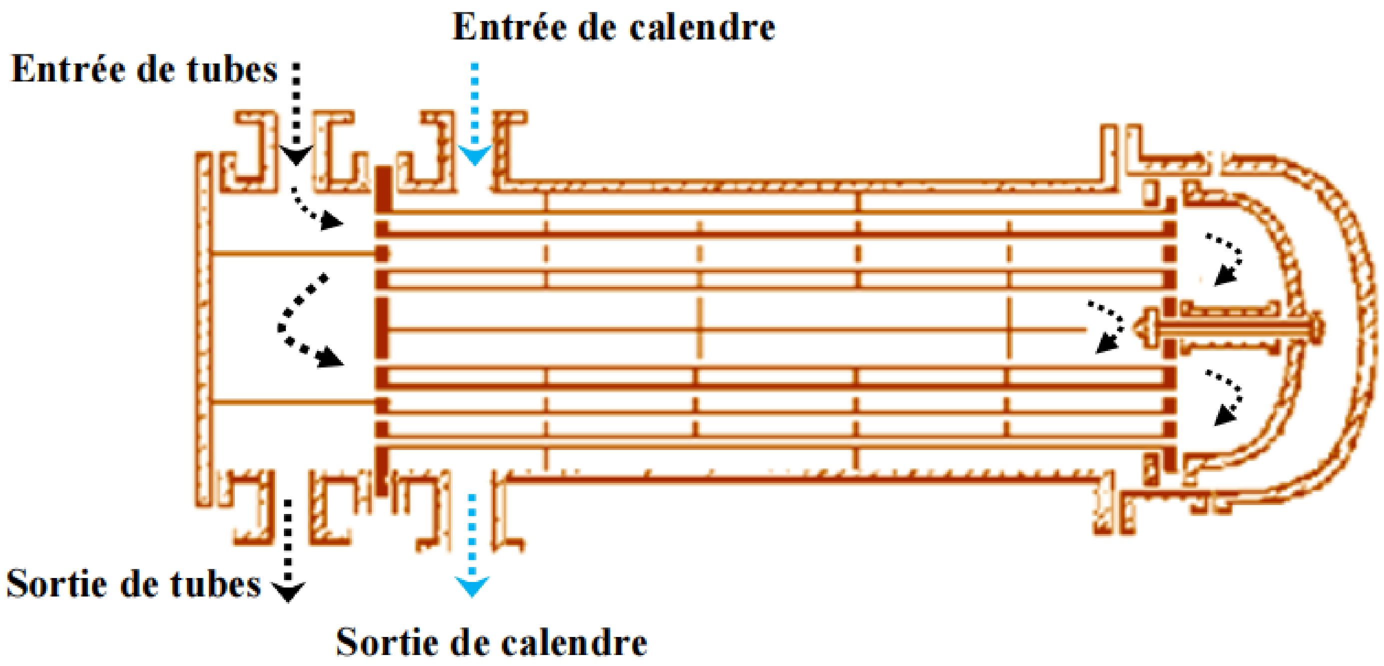


Figure 6: Principle of a 2-4 heat exchanger.[30]

3.3 Crossflow Heat Exchangers

In this type of exchanger, one of the fluids flows through a series of tubes, while the other flows perpendicularly around the tubes. Generally, the liquid circulates inside the tubes and the gas flows outside. The tubes are equipped with fins to enhance the heat transfer by increasing the exchange surface area. The radiator used for cooling in motor vehicles is an example of this type of heat exchanger.

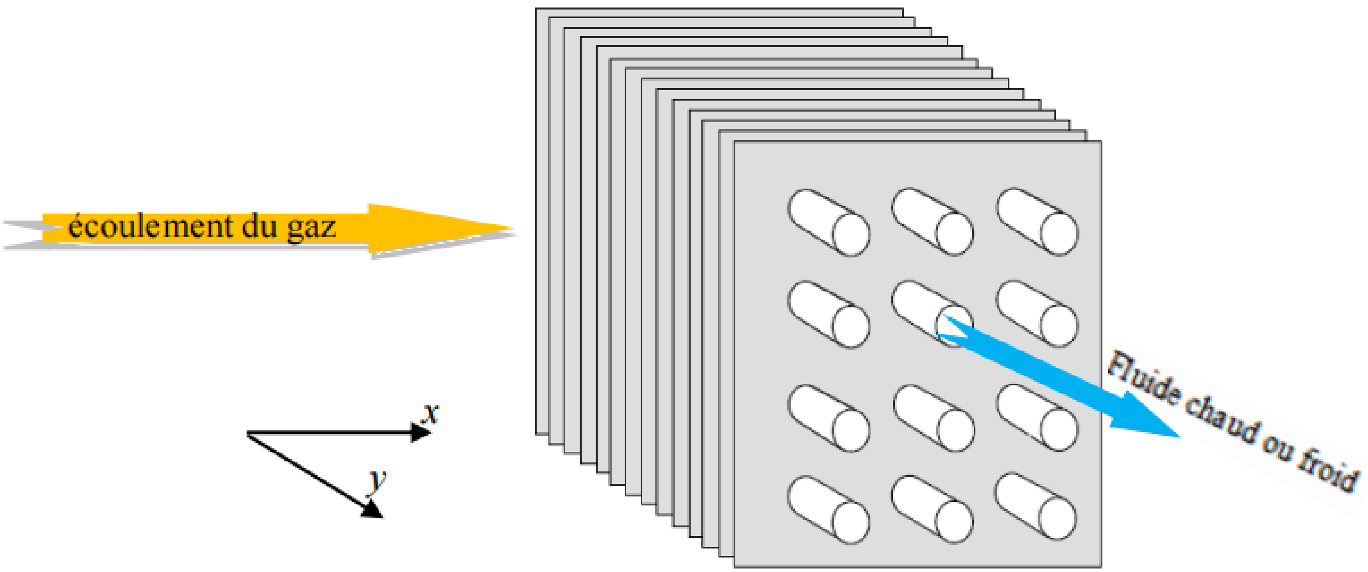
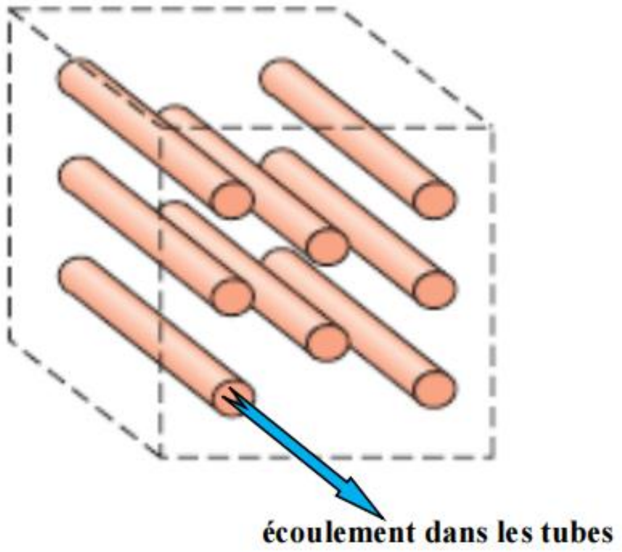


Figure 7: Cross-flow exchangers, unmixed fluids.[9, 11]



Figures 8: Cross-flow exchanger, single mixed fluid. [11]

3.4 Plate Heat Exchangers

This type of heat exchanger consists of a set of plates assembled in such a way that the fluid can flow between them. A series of gaskets ensures the distribution of the fluids between the plates so that each of the two fluids alternately flows through successive inter-plate spaces.

Heat exchange between the fluids takes place through the plates. The compactness of this type of exchanger is a significant advantage, as it allows for a large heat transfer surface area within a limited volume. Therefore, its usefulness becomes particularly evident for high-power applications.

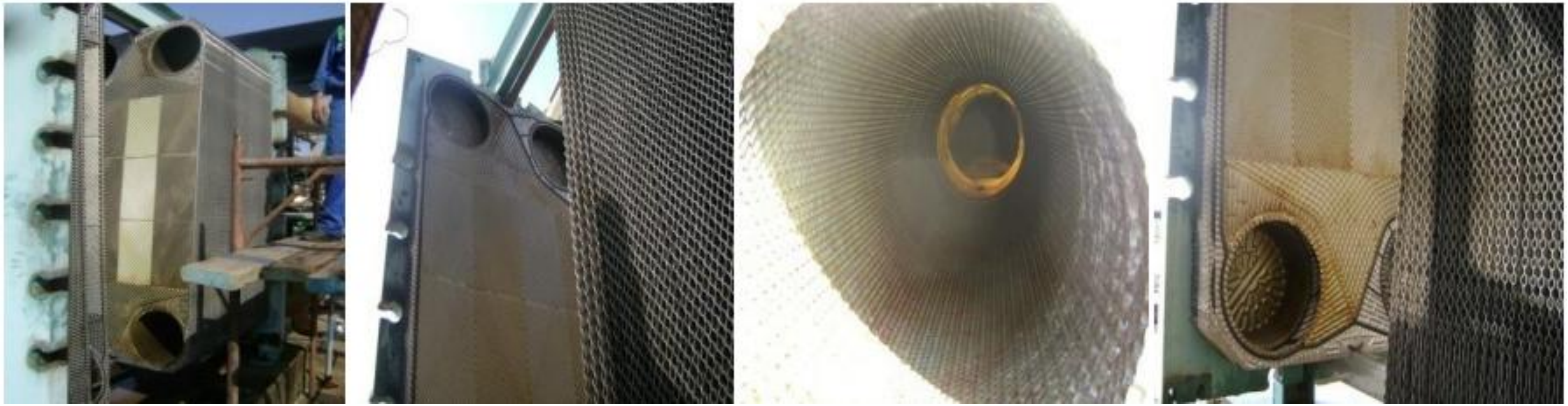


Figure 9: Example of the different parts of a plate exchanger. (CP/2K, Sonatrach of Skikda)

Plate heat exchangers are widely used in the food processing industry, as well as in the nuclear and chemical industries. For obvious reasons of hygiene and public health, the plates are generally made of stainless steel.

It should be noted that the operating temperature range is limited by the use of gaskets made from organic materials.

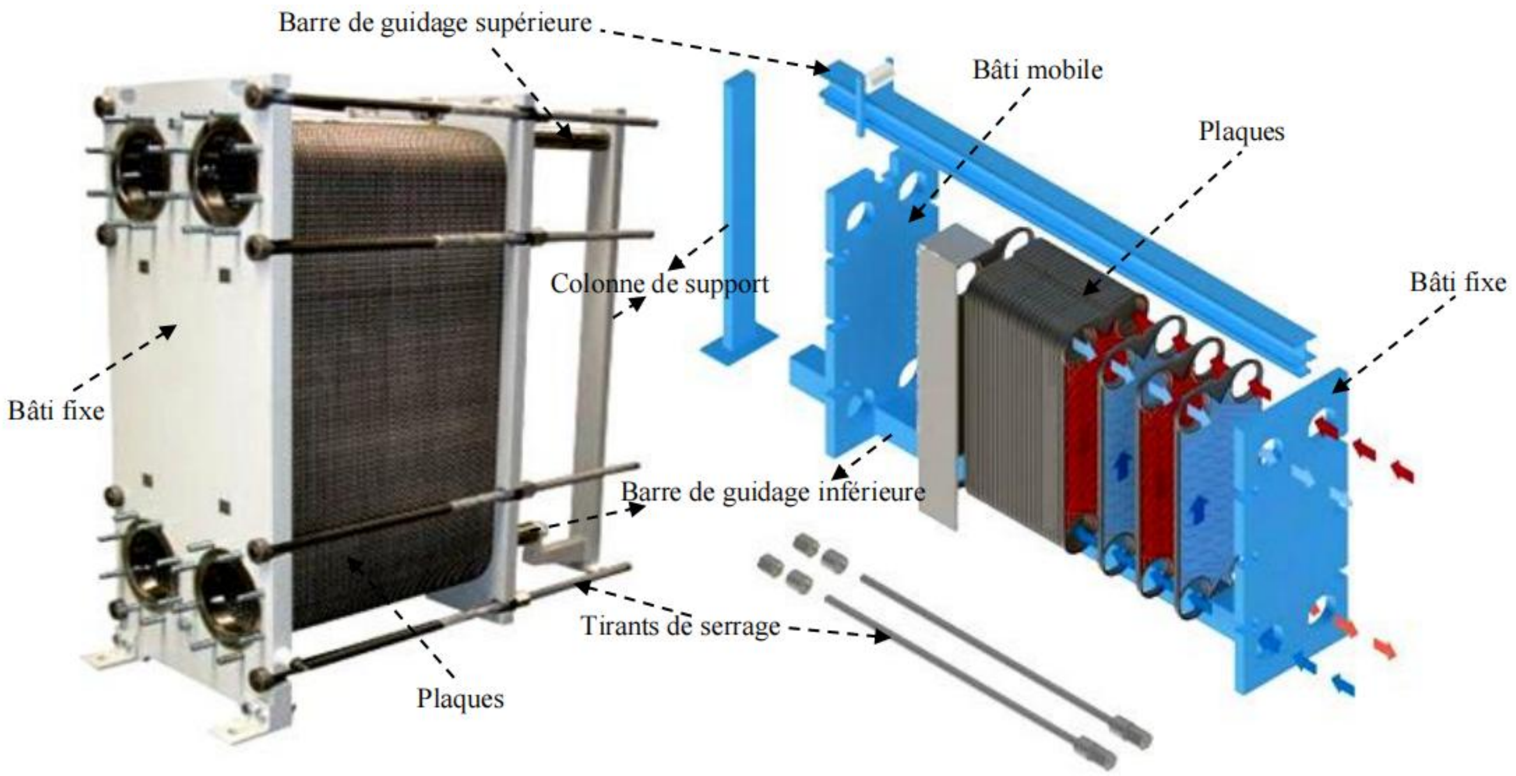


Figure 10: Principle and components of a gasketed plate heat exchanger. [25]

Thus, other classifications of this type of heat exchanger (compact/plate type) can be found, such as:

- Spiral plate heat exchangers.

- Circular plate heat exchangers.

- Welded plate heat exchangers.

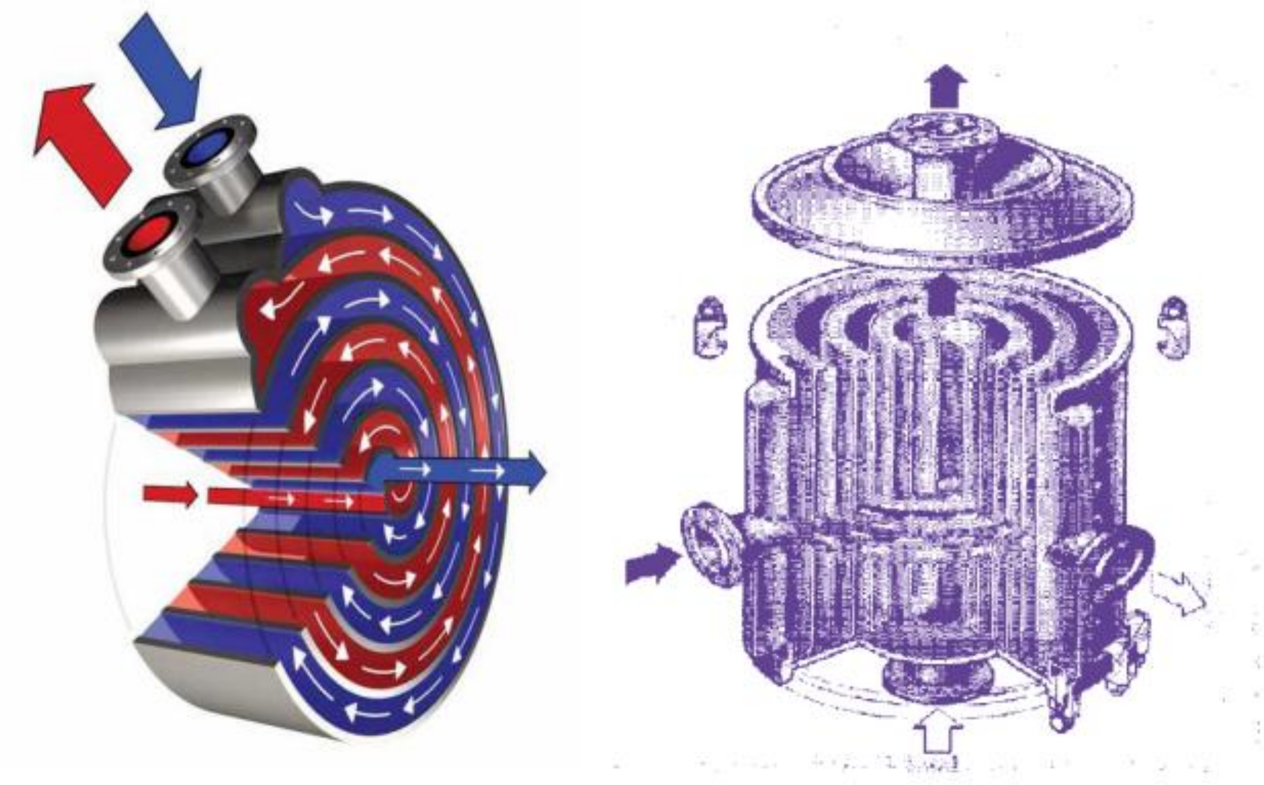


Figure 11: Spiral plate heat exchanger. The two fluids circulate in countercurrent [25]

The spiral heat exchanger (Figure 11) is a plate-type exchanger consisting of two metal sheets wound in parallel. On one hand, one of the fluids flows from the periphery toward the center of the device and from the center to the opposite periphery (the outlet). On the other hand, the second fluid, which is often the primary fluid, flows directly and perpendicularly through the exchanger.

Table 1: Characteristics of the plate exchanger shown in figure 9

Echangeur à plaque 410-440		Côté enveloppe		Côté tube	
Fluide circulant		Eau de mer		Eau dessalée	
Débit total	kg/h	7 725 000		3 379 799	
		Entrée	Sortie	Entrée	Sortie
Débit d'eau	kg/h	7 725 000	7 725 000	3 379 799	3 379 799
Densité du liquide	kg/m^3	1030	1030	993	996
Viscosité du liquide	cp	0.89	0.81	0.67	0.82
Enthalpie du liquide	kcal/kg	25.02	29.2	38.68	29
Chaleur spécifique	kcal/kg°c	0.998	0.998	0.998	0.998
Conductivité thermique	kcal/h.m. °c	0.52	0.53	0.54	0.53
Température service	°c	25	29.2	38.7	29
Pression de service	kg/cm^2g	1.7	-	7.25	-
Chute de pression totale	kg/cm^2	1		1	
Coefficient d'encrassement	$°c.m^2/kcal$	0.00005		0.00004	
Chaleur échangée	kcal/h	32 705 800			
Pression design	kg/cm^2g	6.4		9.6	
Température design	°c	55		64	

Generally, several classifications of heat exchangers can be found, such as the one presented in Table 2.

Table 2: Classification of heat exchangers. [26]

Selon	Type des échangeurs de chaleur	Exemples
Procédé du transfert	Contact direct	
	Contact indirect	a) transfert direct b) stockage c) lie fluidisé
Compacité de la surface	Compacte	
	Non compacte	
Construction	Tubulaire	a) double pipe b) tube et calandre c) tube spiral
	A plaque	a) à joint b) spiral, c) lamelle
	Surface étendue	a) plaque ailettée, b) tube ailetté,
	Régénérative	a) disque rotatif b) tambour rotatif c) matrice fixée

<table>
<tr><td rowspan="2">Arrangement de l'écoulement</td><td>Un passe</td><td>a) écoulement parallèle
b) contre écoulement
c) écoulement croisé</td></tr>
<tr><td>Multi passe</td><td>a) surface étendue à travers un contre écoulement,
b) surface étendue à travers un écoulement parallèle,
c) calandre et tube en écoulement parallèle /contre courant mélangé,
d) calandre et tube en écoulement séparé,
e) calandre et tube en plaque d'écoulement divisé</td></tr>
<tr><td rowspan="3">Nombre de fluides</td><td colspan="2">Deux fluides</td></tr>
<tr><td colspan="2">Trois fluides</td></tr>
<tr><td colspan="2">Plusieurs fluides</td></tr>
<tr><td rowspan="4">Transfert de chaleur</td><td colspan="2">Mécanisme de convection dans une phase sur les côtés</td></tr>
<tr><td colspan="2">Convection d'une phase d'un côté et convection de deux phase à l'autre</td></tr>
<tr><td colspan="2">Convection de deux phases sur les deux côtés</td></tr>
<tr><td colspan="2">Combinaison du transfert de chaleur convectif et radiatif</td></tr>
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4. Calculation of Heat Exchangers

For the cooling or heating of fluids, certain processes require the use of intermediate heat transfer or refrigerant fluids.

The parameters necessary for the design of a heat exchanger are:

- **The exchange surface** (surface of the plates, inner surface of concentric tubes, etc.): **A** in **m^2**
- **The characteristics of the device** (thermal conductivity, plate thickness) and of the **fluid** (viscosity, laminar or turbulent regime, etc.), which determine the **overall heat transfer coefficient U** in **$W/(m^2 \cdot °C)$** or **$W/(m^2 \cdot K)$**
- **The temperature difference** between one side and the exchange surface on the other side. The **logarithmic mean temperature difference** between the ends of the exchanger is used, denoted **ΔT (°C)**

4.1. Notations

In what follows, the following assumptions are used:

- The regime is steady: all parameters and variables are constant over time.
- The exchanger is adiabatic (signifie qu’il n’y a aucun transfert de chaleur entre l’échangeur et son environnement extérieur : la chaleur ne “sort” ni ne “rentre” depuis l’extérieur pendant le fonctionnement. En d’autres termes, tout l’échange de chaleur se produit uniquement à l’intérieur de l’échangeur, sans perte vers l'extérieur).
- The thermo-physical properties of the fluids remain constant within the considered temperature ranges.
- The temperatures are one-dimensional and vary only in one direction of flow.
- There are no pressure (head) losses during the flow.

The performance of a heat exchanger is mainly evaluated by its overall thermal conductance. This conductance can be calculated based on the geometry, the properties of the wall, and the properties of both hot and cold fluids, as well as on the operating conditions (mass flow rates and inlet temperatures of both fluids).

The calculation approach is simple but necessary. It enables students to become familiar with the computation of heat exchanger performance.

It consists in calculating **the Reynolds and Prandtl numbers** for each fluid, evaluating the **internal and external convection coefficients** based on the available Nusselt number correlations related to internal flows, and finally calculating the **overall heat transfer coefficient**.

4.2 Overall Heat Transfer Coefficient, *U*

Determining an **overall heat transfer coefficient U**, is one of the most uncertain aspects of a heat exchanger. This uncertainty arises from the continuous degradation of the exchanger. It is defined by the following relation:

$$d\Phi = U(T_c - T_f)dA$$

In a heat exchanger, the heat transfer from the hot fluid to the cold fluid essentially results from the following phenomena:

- Convection between the hot fluid and the outer surface of the solid wall.
- Conduction through this solid wall.
- Convection between the inner surface of the solid wall and the cold fluid.

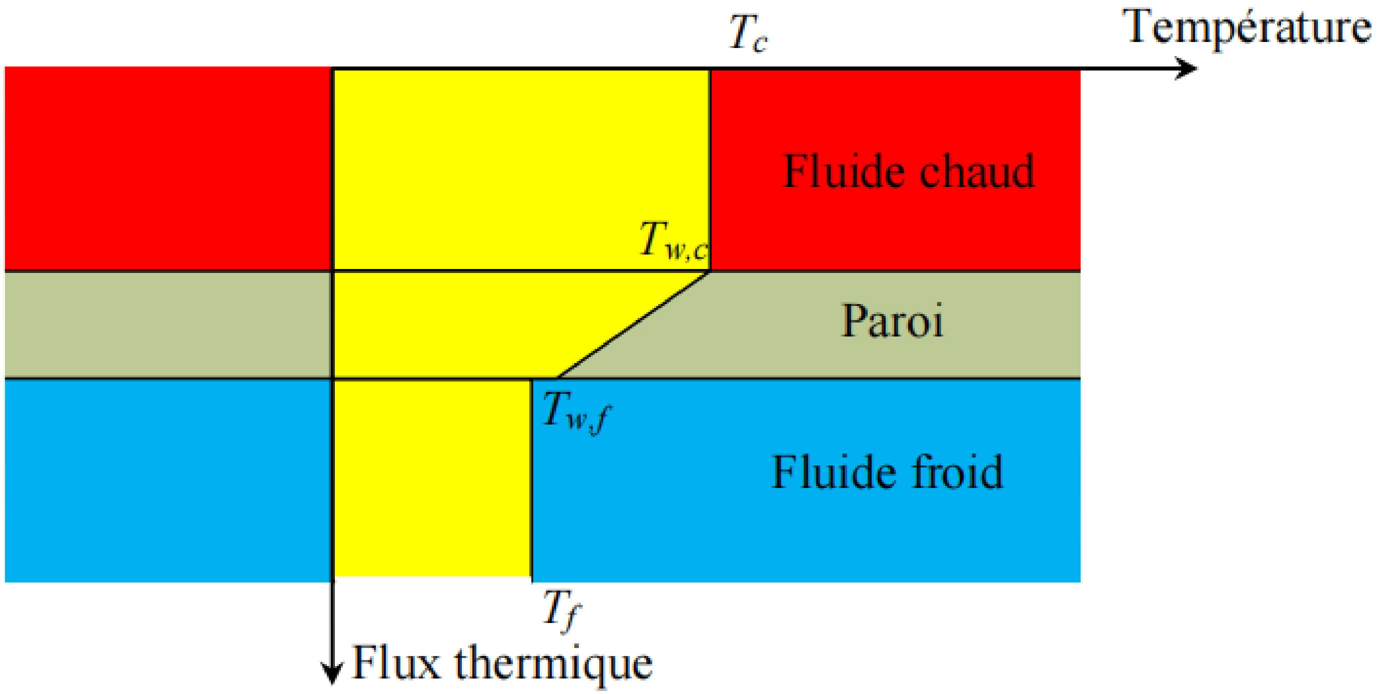

Figure 12: Temperature profile across a surface element dA.

The convective thermal resistance of the hot fluid is: $\frac{1}{h_c\ A}$

Where : h_c is the convection coefficient of the hot fluid.

The convective thermal resistance of the cold fluid is: $\frac{1}{h_f\ A}$

Where: h_f is the convection coefficient of the cold fluid.

The conductive thermal resistance through the solid wall of thickness e and thermal conductivity λ is: $\frac{e}{\lambda\, A}$

The expression for the heat flux transferred from the hot fluid to the cold fluid is:

$$\Phi = \frac{T_c - T_f}{\frac{1}{h_c A} + \frac{e}{\lambda A} + \frac{1}{h_f A}} \qquad \textbf{(2)}$$

or equivalently :

$$\Phi = UA(T_c - T_f)$$

Where $\boldsymbol{U}$ is the overall heat transfer coefficient:

$$\frac{1}{U} = \frac{1}{h_c} + \frac{e}{\lambda} + \frac{1}{h_f} \qquad \textbf{(3)}$$

To more accurately represent the phenomena occurring in a real heat exchanger, the following factors must be taken into account:

In relation (3), the heat transfer surface area $\boldsymbol{A}$ on the hot side and cold side is assumed to be the same. However, in practice, the extent of the exchange surface in contact with the two fluids is not always identical. It is therefore necessary to introduce two surface areas, $\boldsymbol{A_{(c)}}$ and $\boldsymbol{A_{(f)}}$, and to express the overall heat transfer coefficient either:

- per unit surface area on the hot side, denoted $\boldsymbol{U_{(c)}}$.
- per unit surface area on the cold side, denoted $\boldsymbol{U_{(f)}}$.

Furthermore, after a certain operating time, the exchange walls become covered with a fouling film. These deposits of scale and dirt have a thermal conductivity much lower than that

1. of the metal and therefore represent additional thermal resistances $\boldsymbol{\ddot{R}_{(c)}}$ and $\boldsymbol{\ddot{R}_{(f)}}$, which hinder heat transfer.

Hence, the performance of the exchanger is determined by calculating one of the following overall heat transfer coefficients:

$$U_c = \frac{1}{\frac{1}{h_c} + \ddot{R}_c + \frac{e}{\lambda}\frac{A_c}{A_m} + \left(\ddot{R}_f + \frac{1}{h_f}\right)\frac{A_c}{A_f}} \quad \textbf{(4)}$$

$$U_f = \frac{1}{\frac{1}{h_f} + \ddot{R}_f + \frac{e}{\lambda}\frac{A_f}{A_m} + \left(\ddot{R}_c + \frac{1}{h_c}\right)\frac{A_f}{A_c}} \quad \textbf{(5)}$$

A_f: area of the heat exchange surface on the cold side, **[m²]**

A_c: area of the heat exchange surface on the hot side, **[m²]**

A_m: mean heat exchange surface area, **[m²]**

R_c'' and R_f'' are the thermal resistances per unit area of the fouling films deposited on the hot and cold sides of the exchange surface, expressed in **(m²·°C)/W**.

U_c and U_f are expressed in **W/(m²·°C)**.

U is defined as a function of the total thermal resistance to heat transfer between the two fluids, and it can be expressed as follows [11]:

$$\frac{1}{UA} = \frac{1}{U_f A_f} = \frac{1}{U_c A_c} = \frac{1}{h_f A_f} + \frac{\ddot{R}_f}{A_f} + R_{paroi} + \frac{\ddot{R}_c}{A_c} + \frac{1}{h_c A_c} \qquad \textbf{(6a)}$$

Furthermore, it is known that fins are often added to one or both of the surfaces exposed to the fluids.
This increases the surface area and reduces the overall thermal resistance to heat transfer.

Consequently, when accounting for the effect of fins (extended surfaces), the overall heat transfer coefficient is modified as follows [11]:

$$\frac{1}{UA} = \frac{1}{(\eta_0 hA)_f} + \frac{\ddot{R}_f}{(\eta_0 A)_f} + R_{paroi} + \frac{\ddot{R}_c}{(\eta_0 A)_c} + \frac{1}{(\eta_0 hA_c)} \qquad \textbf{(6b)}$$

The term $\boldsymbol{\eta_o}$ in Equation (6b) represents the overall efficiency of the finned surface.

The heat transfer rate for the hot or cold surface is given by:

$$\Phi = \eta_0 hA(T_b - T_\infty) \qquad \textbf{(7)}$$

where $\mathbf{T_b}$ is the *base surface temperature* and **A** is the *total surface area* [11].

The overall finned surface efficiency η_0 is expressed as:

$$\eta_0 = 1 - \frac{A_f}{A}(1 - \eta_\xi) \qquad \textbf{(8)}$$

where:

$\mathbf{A_\xi}$ is the *total finned surface area.*

$\boldsymbol{\eta_\xi}$ is the *efficiency of a single fin***.**

4.2.1 Fouling Resistances, *R¨*

The values of the fouling resistances (Table 3) are determined from comparative measurements taken between the initial operating conditions and those observed over time during operation.

These values typically range from **1×10^{-4} to 70×10^{-4} ($m^2\cdot°C$)/W**.

Table 3: Some fouling resistance values. [5, 9, 11]

Fluides	Valeurs moyennes de $\ddot{R}$ [m^2°C/W]
Eau de mer à T < 50°C	$9\ 10^{-5}$
Eau de mer à T > 50°C	$2\ 10^{-4}$
Eau de ville à T < 50°C	$2\ 10^{-4}$
Eau de ville à T > 50°C	$3.5\ 10^{-4}$
Eau de rivière	3.5 à 7 10^{-4}
Vapeur d'eau non grasse	$9\ 10^{-5}$
Vapeur d'eau grasse	$2\ 10^{-4}$
Liquides réfrigérants	1.8 à 2 10^{-4}
Fioul	4 à 9 10^{-4}
Essence, kérosène, gas-oil	$2\ 10^{-4}$
Huile de lubrification	$1.8\ 10^{-4}$
Air non dépoussiéré	$3.5\ 10^{-4}$
Air industrielle	$4\ 10^{-4}$
Produits de combustion gazeux	20 à 70 10^{-4}
Vapeurs d'Alcool	$9\ 10^{-5}$

The average values of the fouling resistance recommended for a plate heat exchanger are approximately ten times lower than those presented in Table (3).

4.2.2 Determination of the Convective Heat Transfer Coefficients $h_{(c)}$ and $h_{(f)}$

- Evaluation of the Prandtl number for each flow, based on the physical properties of the fluid considered**:**

$$Pr = \frac{\mu C_p}{\lambda} \quad (9)$$

where:

***μ*: dynamic viscosity, [kg/(m·s)]**

***C_p*: specific heat at constant pressure, [J/(kg·°C)]**

***λ*: thermal conductivity, [W/(m·°C)]**

Calculation of the Reynolds number for each flow **[9]**

$$Re = \frac{\rho v d_h}{\mu} \quad (10)$$

ρv : is the **mass velocity** of the fluid, [kg/(m²·s)]

$$\rho v = \frac{\dot{m}}{A}$$

where: $\dot{m}$: mass flow rate, [kg/s]

A: flow cross-sectional area, [m²]

$\boldsymbol{d_h}$ is the **hydraulic diameter**, [m], defined as: $$\boldsymbol{d_h = \frac{4A}{p}} \qquad (11)$$

(This formula is particularly useful for non-circular sections).

where:

A: cross-sectional area of the fluid passage, [m²]

p: wetted perimeter, [m]

Note:

1. **In the case of a circular pipe, the hydraulic diameter is equal to the actual diameter** $(\boldsymbol{d_h = d})$
2. The calculation of the Reynolds number for each fluid is necessary to determine whether the flow is **laminar** or **turbulent**, and to select the appropriate empirical correlation for the problem.

❑ For each flow, the **Nusselt number** is calculated as:

$$Nu = \frac{h\, d_h}{\lambda} \qquad (12)$$

After determining the Prandtl and Reynolds numbers, the Nusselt number *Nu* can easily be obtained using experimental (empirical) correlations ; $\boldsymbol{Nu = f(Re, Pr)}$; found in the literature (see Chapter 2).

The calculation of the Nusselt number makes it possible to determine the desired **convective heat transfer coefficient** h.

The heat transfer performance of an exchanger can be improved by:

- Varying the temperature along the exchanger
- Adding **baffles** in the shell (for shell-and-tube exchangers, baffles make the shell-side fluid flow turbulent),

Using **finned tubes**, etc.

4.3 LMTD Method (Log Mean Temperature Difference)

There are two main types of tubular heat exchangers: the **parallel-flow (co-current) exchanger** and the **counter-flow exchanger**.

The main objective is to **minimize the heat transfer surface area and pressure drop** for a given rate of heat transfer.

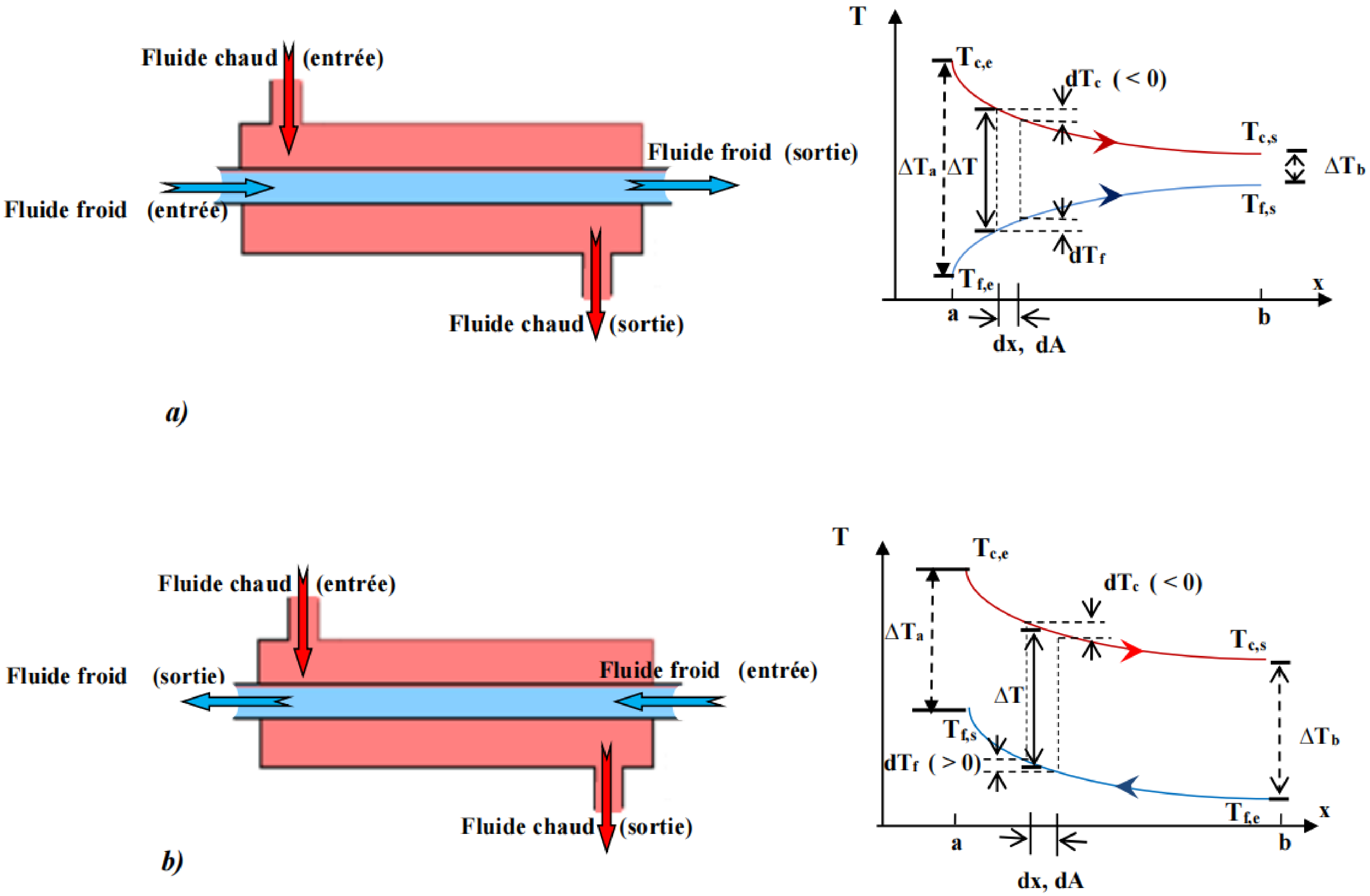

Figure 13: Operating principle of exchangers **a) EACP and b) EACC**.

- Based on the two figures above, reversing the flow direction of one of the fluids makes the **heat transfer in the counter-flow exchanger more significant** than that in the **parallel-flow (co-current) exchanger** ?

Yes — reversing the direction of flow of one of the fluids (going from “co-current” to “counter-current”) generally makes the heat transfer greater for the same overall coefficient U and the same exchange surface A.

- The Exchanged Heat Flux:

$$\Phi = \dot{m}(\bar{h}_2 - \bar{h}_1) = \dot{m}C_p(T_2 - T_1) \qquad (13)$$

$$\Phi = \dot{m}_c(\bar{h}_{c,e} - \bar{h}_{c,s}) = \dot{m}_c C_{p,c}(T_{c,e} - T_{c,s}) \qquad (14)$$

$$\Phi = \dot{m}_f(\bar{h}_{f,s} - \bar{h}_{f,e}) = \dot{m}_f C_{p,f}(T_{f,s} - T_{f,e}) \qquad (15)$$

Assuming there is **no heat loss**, the **energy balance** for a counterflow heat exchanger through a surface element $\boldsymbol{dA}$ of length $\boldsymbol{dx}$ is expressed as follows:

$$d\Phi = U\,dA\,(T_c - T_f) = -\dot{m}_c C_{p,c}\,dT_c, dT_c < 0 = \dot{m}_f C_{p,f}\,dT_f, dT_f > 0 \qquad (16)$$

Therefore:

$$dT_c = -\frac{d\Phi}{\dot{m}_c C_{p,c}} \qquad (17)$$

$$dT_f = \frac{d\Phi}{\dot{m}_f C_{p,f}} \qquad (18)$$

Hence,

$$d(T_c - T_f) = -d\Phi\left(\frac{1}{\dot{m}_c C_{p,c}} + \frac{1}{\dot{m}_f C_{p,f}}\right) \qquad (19)$$

4.3.1 Parallel-flow heat exchanger (PFE)

From equations (16) and (19) we eliminate $d\Phi$:

$$\frac{d(T_c - T_f)}{(T_c - T_f)} = -U\, dA(\frac{1}{\dot{m}_c C_{p,c}} + \frac{1}{\dot{m}_f C_{p,f}}) \qquad (20)$$

Or equivalently

$$\frac{d\Delta T}{\Delta T} = -U(\frac{1}{C_c} + \frac{1}{C_f})\, dA \qquad (21)$$

with $C_c = \dot{m}_c C_{p,c}$, $C_f = \dot{m}_f C_{p,f}$, $\Delta T = (T_c - T_f)$, $(\Delta T)_{x=0} = \Delta T_a$, $(\Delta T)_{x=L} = \Delta T_b$.

If ***U*** **is constant** along the exchanger then

$$\int_a^b \frac{d\Delta T}{\Delta T} = -U(\frac{1}{C_c} + \frac{1}{C_f}) \int_a^b dA \Rightarrow \ln\frac{\Delta T_b}{\Delta T_a} = -UA(\frac{1}{\dot{m}_c C_{p,c}} + \frac{1}{\dot{m}_f C_{p,f}})$$

Hence:

$$\ln\frac{\left(T_{c,s}-T_{f,s}\right)}{\left(T_{c,e}-T_{f,e}\right)}=-UA\left(\frac{1}{\dot{m}_c C_{p,c}}+\frac{1}{\dot{m}_f C_{p,f}}\right) \qquad (22)$$

But:

$$\Phi = C_c\ (T_{c,e}-T_{c,s}) = C_f\ (T_{f,s}-T_{f,e}) \qquad (23)$$

Therefore equation (22) can be written as:

$$\Phi = UA\ \frac{(T_{c,s}-T_{f,s})-(T_{c,e}-T_{f,e})}{\ln\left(\frac{T_{c,s}-T_{f,s}}{T_{c,e}-T_{f,e}}\right)} = UA\ \frac{\Delta T_b-\Delta T_a}{\ln\left(\frac{\Delta T_b}{\Delta T_a}\right)} \qquad (24)$$

4.3.2 Counter-flow heat exchanger (CFHE)

For a counter-flow heat exchanger (CFHE), $\boldsymbol{dT_f < 0}$ in the direction of positive x.

Then

$$d\Phi = -\dot{m}_c C_{p,c}\, dT_c = -\dot{m}_f C_{p,f}\, dT_f \qquad (25)$$

From equation (19):

$$(T_c - T_f) = -d\Phi\left(\frac{1}{\dot{m}_c C_{p,c}} + \frac{1}{\dot{m}_f C_{p,f}}\right)$$

and in the same way as for the parallel-flow exchanger (PFE), we eliminate $d\Phi$from equations (16) and (19):

$d\Phi = U\, dA\, (T_c - T_f)$, thus

$$\frac{d(T_c - T_f)}{(T_c - T_f)} = -U\, dA\left(\frac{1}{\dot{m}_c C_{p,c}} + \frac{1}{\dot{m}_f C_{p,f}}\right) \qquad (26)$$

By integrating for constant U:

$$\ln\left(\frac{T_{c,s} - T_{f,e}}{T_{c,e} - T_{f,s}}\right) = -UA\left(\frac{1}{\dot{m}_c C_{p,c}} - \frac{1}{\dot{m}_f C_{p,f}}\right) \qquad (27)$$

Finally:

$$\Phi = UA \frac{(T_{c,e} - T_{f,s}) - (T_{c,s} - T_{f,e})}{\ln\left(\frac{T_{c,e} - T_{f,s}}{T_{c,s} - T_{f,e}}\right)} = UA \frac{\Delta T_a - \Delta T_b}{\ln\left(\frac{\Delta T_a}{\Delta T_b}\right)} = UA\, \Delta T_{\mathrm{lm}} \qquad (28)$$

where

ΔT_{lm} = Log Mean Temperature Difference (LMTD)

Remarks:

- The **LMTD method** can be used if the inlet and outlet temperatures (T_e and T_s) are known or can be determined.
- If only the **inlet temperatures** are known, the LMTD method requires an **iterative procedure**. In such a case, it is preferable to use the **effectiveness–NTU method**.
- In a **counter-flow operation**, it is possible to have $T_{f,s} > T_{c,s}$. However, it is **impossible** to obtain $T_{f,s} > T_{c,e}$or $T_{c,e} < T_{f,s}$.

4.3.3 Comparison between Parallel-flow and Counter-flow Heat Exchangers

For a **counter-flow tubular heat exchanger**, the **log mean temperature difference (LMTD)** is higher, which results in a **greater heat transfer rate**.

Exercise 1:

Counter-flow and parallel-flow heat exchangers are subjected to the following conditions:

$$T_{c,e} = 100°C, T_{c,s} = 45°C$$

$T_{f,e} = 15°C, T_{f,s} = 30°C$- **Calculate their Log Mean Temperature Differences (LMTD)** ΔT_{lm}.

Exercise 1:

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Solution :

Contre-courant

$$\Delta T_{LM} = \frac{(T_{c,e} - T_{f,s}) - (T_{c,s} - T_{f,e})}{ln\left(\frac{T_{c,e} - T_{f,s}}{T_{c,s} - T_{f,e}}\right)} = \frac{(100 - 30) - (45 - 15)}{ln\left(\frac{100 - 30}{45 - 15}\right)} = 47{,}20\ °C$$

Co-courant

$$\Delta T_{LM} = \frac{(T_{c,s} - T_{f,s}) - (T_{c,e} - T_{f,e})}{ln\left(\frac{T_{c,s} - T_{f,s}}{T_{c,e} - T_{f,e}}\right)} = \frac{(45 - 30) - (100 - 15)}{ln\left(\frac{45 - 30}{100 - 15}\right)} = 40{,}35\ °C$$

Exercise 2:

Counter-flow and parallel-flow heat exchangers are subjected to the following conditions:

$$T_{c,e} = 90°C, T_{c,s} = 35°C$$

$T_{f,e} = 20°C, T_{f,s} = 30°C$ **Calculate their Log Mean Temperature Differences (LMTD).**

Solution :

Contre-courant

$$\Delta T_{LM} = \frac{(T_{c,e} - T_{f,s}) - (T_{c,s} - T_{f,e})}{ln\left(\frac{T_{c,e} - T_{f,s}}{T_{c,s} - T_{f,e}}\right)} = \frac{(90 - 30) - (35 - 20)}{ln\left(\frac{90 - 30}{35 - 20}\right)} = 32,46\ °C$$

Co-courant

$$\Delta T_{LM} = \frac{(T_{c,s} - T_{f,s}) - (T_{c,e} - T_{f,e})}{ln\left(\frac{T_{c,s} - T_{f,s}}{T_{c,e} - T_{f,e}}\right)} = \frac{(35 - 30) - (90 - 20)}{ln\left(\frac{35 - 30}{90 - 20}\right)} = 24,63\ °C$$

Therefore, it is preferable to choose a counter-current operation. In general, a heat exchanger of any configuration will always have better performance than a simple parallel-flow tubular exchanger and lower performance than a simple counter-current tubular exchanger. This conclusion is clearly illustrated in Figure 13.

Exercise 3

Parallel-flow and counter-flow heat exchangers are subjected to the following conditions:

$$T_{c,e} = 110°C,\ T_{c,s} = 30°C\ ;\ \dot{m}_c = 5000\ \text{kg/h}\ ;\quad C_{p.c} = 2100\ J/kgK$$

$$T_{f,e} = 12°C,\ T_{f,s} = ?\ °C\ ;\quad \dot{m}_f = 12000\ \text{kg/h}\ ;\ C_{p.f} = 4180\ J/kgK$$

1. Calculate their heat transfer areas.
2. Calculate the heat duty (power) of the counter-flow exchaner

Solution :

$$\Phi = \dot{m}_c C_{p,c}\left(T_{c,e} - T_{c,s}\right)$$

$$\Phi = \dot{m}_f C_{p,f}\left(T_{f,s} - T_{f,e}\right)$$

$$\Phi = (1{,}38)(2100)(110 - 30) = 2{,}3\ 10^5\ \text{W}$$

$$\dot{m}_f C_{p,f}\left(T_{f,s} - T_{f,e}\right) = \dot{m}_c C_{p,c}\left(T_{c,e} - T_{c,s}\right) = 2{,}3\ 10^5\ \text{W}$$

$$T_{f,s} = \frac{\Phi}{\dot{m}_f C_{p,f}} + T_{f,e} = \frac{2{,}3\ 10^5}{3{,}33.4180} + 12 = 28{,}52\ °C$$

➢ Co-courant (EACP)

$$\Phi = UA\frac{(T_{c,s} - T_{f,s}) - (T_{c,e} - T_{f,e})}{ln\left(\frac{T_{c,s} - T_{f,s}}{T_{c,e} - T_{f,e}}\right)} = UA\frac{\Delta T_b - \Delta T_a}{ln\left(\frac{\Delta T_b}{\Delta T_a}\right)}$$

$$=> \ \mathrm{A} = \Phi\frac{ln\left(\frac{\Delta T_b}{\Delta T_a}\right)}{(\Delta T_b - \Delta T_a)U} = 2{,}3\ 10^5\ \frac{ln\left(\frac{1{,}48}{98}\right)}{(1{,}48 - 98)300}$$

$\boldsymbol{A} = \mathbf{33{,}35}\ \mathrm{m}^2$ **…………** heat transfer areas (**case co- courant)**

- **Contre-courant (EACC)**

$$\Phi = UA\frac{(T_{c,e} - T_{f,s}) - (T_{c,s} - T_{f,e})}{ln\left(\frac{T_{c,e} - T_{f,s}}{T_{c,s} - T_{f,e}}\right)} = -U\,\mathrm{A}\,\Delta T_{LM} = UA\frac{\Delta T_a - \Delta T_b}{ln\left(\frac{\Delta T_a}{\Delta T_b}\right)}$$

$$\Delta T_b = T_{c,s} - T_{f,e} = 30 - 12 = 18\,°C$$

$$\Delta T_a = T_{c,e} - T_{f,s} = 110 - 28{,}52 = 81{,}48\,°C$$

$$=> \; A = \Phi\frac{ln\left(\frac{\Delta T_b}{\Delta T_a}\right)}{(\Delta T_b - \Delta T_a)U} = 2{,}3\;10^5\;\frac{ln\left(\frac{81{,}48}{18}\right)}{(81{,}48 - 18)300} = 18{,}23\;\mathrm{m}^2$$

A=18,23 m^2 **…………** heat transfer areas (**case contre- courant)**