II. THE GRAVIMETRIC METHOD

II.1 Introduction

Gravimetry is a geophysical technique that measures variations in the Earth's gravitational potential field. It is a prospecting method that allows the detection of density anomalies in the subsurface. These minute variations, caused by density contrasts in the Earth's crustal constituents, can be measured by gravimetric surveys using highly sensitive instruments.

Geophysical and Environmental Applications

Mineral and Oil Exploration: Detecting areas of abnormal density that may correspond to ore veins (denser mass) or oil/gas deposits (less dense mass).

Cavity Detection: Identifying areas of under-density caused by underground cavities, such as caves.

Water Cycle Study: Monitoring variations in groundwater reserves, related to precipitation and evaporation, through repeated measurements over time.

Dynamic Monitoring: Monitoring Earth-scale phenomena, such as ice melt, groundwater movements, or volcanic activity (swelling due to magma).

Earth Structure Study: Understanding the planet's internal structure by analyzing large gravitational anomalies due to density heterogeneities in the crust and mantle.

II.2.1 Basic Principle

II.2.1.1 Laws of Universal Gravitation

Newton's First Law

Two particles of mass m1 and m2 separated by a distance r are attracted to each other

by a force F such that: $\vec{F} = -\frac{Gm_1m_2}{R^2}\vec{r_1}$

Or **F** is the force applied to the mass $\boldsymbol{m_2}$, $\vec{r_1}$ is the unit vector r_1, is the distance between the masses m_1 rt m_2 , and G is the universal gravitational constant. and G are given by:

$$|\vec{r_1}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$G = 6.67x10^{-8} dyne\ cm^2/g^2 \text{ CGS}$$
$$G = 6.67x10^{-11} Nm^2/Kg^2 \ SI$$

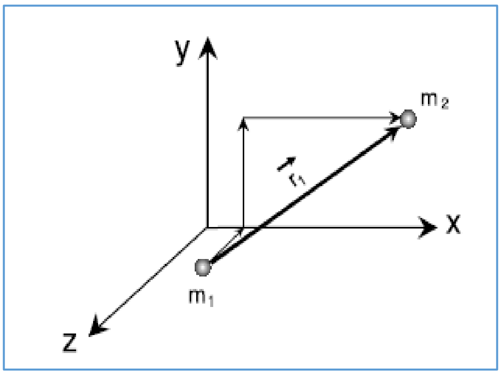


Figure II.1: A Representation of the two point masses m1 and m2 separated by a distance r (After Telford et al. 1990).

Newton's Second Law

A force **F** must be applied to a mass m to cause it to experience an acceleration $\boldsymbol{a}$. This is expressed by the following relationship:

$$\vec{F} = m\vec{a}$$

The acceleration of a mass m at the surface of the ground is therefore expressed by:

$$\vec{a} = -\frac{GM_T}{R_T^2}\vec{r} = \vec{g}$$

$\boldsymbol{g}$ is called "acceleration due to gravity" and is on average 9.81 m/s^2.

For measuring the intensity of the gravitational field, geophysicists frequently use the Gal: 1 Gal = 10^{-2} m/s². It was named in **honor of Galileo**.

Also defined are: the milligal: **1 mGal** = 10^{-5} m/s² and the **microga**l: 1 μGal = 10^{-8} m/s².

- Earth's radius: R_T = 6370 km
- Earth's mass: $M_T = 5.974 \times 10^{24}$ kg
- Universal gravitational constant: **G** = 6.673×10^{-11} m³ kg^{-1} s^{-2}
- Volume of a sphere of radius: $V = \frac{4}{3}\pi R^3$

II.3 Gravitational Potential الإمكانات الجاذبية

The gravitational field المجال الجاذبي is a conservative field, meaning that the work done to move a mass within this field is independent of the path taken. It depends only on the starting and ending points. Therefore, if we return to the starting point, the energy expenditure is zero.

The force that generates a ***conservative field*** can be derived from the scalar function of the potential by:

$$\nabla U = \vec{g} = \frac{\vec{F}}{m_2} \quad ou\ l'opérateur \nabla\ est\ donné\ par:\ \nabla U = \frac{\partial U}{\partial x}\vec{i} + \frac{\partial U}{\partial y}\vec{j} + \frac{\partial U}{\partial z}\vec{k}$$

The potential equation therefore gives us (m2: unit mass):

$$U = \int_{\infty}^{R} \vec{g}\,.\vec{r}dr = -Gm_1 \int_{\infty}^{R} \frac{dr}{r^2} = Gm_1\left[\frac{1}{r}\right] = \frac{Gm_1}{R}$$

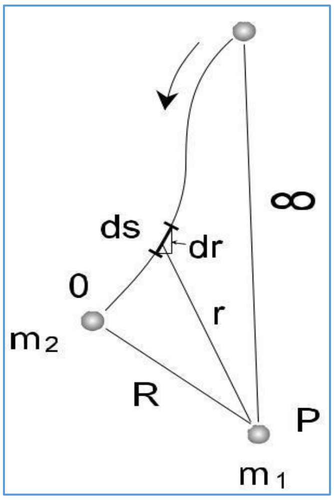


Figure II.2: Gravitational potential between two points

If we have a continuous mass distribution in a volume V outside the point, the potential U at point P is: $U = G\int_V \frac{\rho}{r}dv$

Where ρ is the density (g/cm3) and dv is the volume element (cm3)

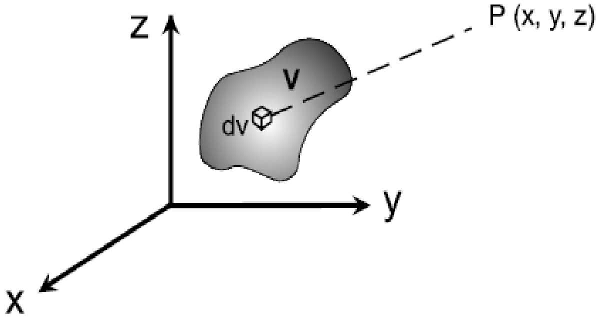


Figure II.3: Distribution of mass in space

II.4 Gravitational Field مجال الجاذبية

Consider a particle at rest at a point A in space. All particles located around the mass m of A undergo acceleration (see figure). Each point in space is then characterized by an acceleration vector. The set of these vectors constitutes the gravitational field of the mass m.

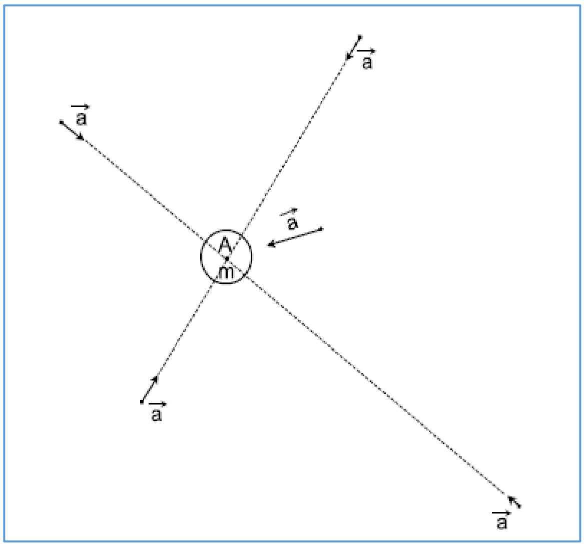


Figure II.4: Gravitational field

II.5 Earth's Shape and Geodesy

Geodesy is the determination of the Earth's "exact" shape from gravimetric measurements.

II.5.1 Theoretical Shape of the Earth and Reference Ellipsoid

- **An ellipsoid of revolution: the spheroid**

  • It is an equipotential surface where, at every point, gravity is normal to the surface.

  • The spheroid is referenced to the mean sea surface, with excess masses removed and oceanic troughs filled.

  • The mathematical expression is a spherical harmonic expansion whose formula, called the International Gravity Formula (IGF), was adopted in 1980 and is written:

$$g = g_0\left(1+\alpha \sin^2 \phi + \beta \sin^2 2\phi\right) \qquad ms^{-2} \quad SI$$
$$avec \ g_0 = 9.780327$$
$$\alpha = 0.0053024$$
$$\beta = -0.0000058$$

The gravity value thus obtained is that which would be observed at sea level on a spheroidal Earth (closely approximating its true shape) whose density varies only with depth and not laterally.

the case if the Earth were spherical and stationary, but varies by nearly 5,000 mGal between the equator (978,000 mGal or 9.78 m.s-2) and the pole (983,000 mGal or 9.83 m.s-2).

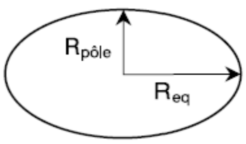
The difference of 5170 mgals between the value at the poles and at the equator is caused by:

1. The effect of the Earth's rotation: The closer you get to the pole, the weaker the centrifugal force, therefore $\vec{g}$ it is at its maximum.

2. The difference between the equatorial radius and the polar radius, i.e., the difference between the Earth's true shape and a sphere. The difference of 5170 mgals is distributed approximately 2/3 for centrifugal force and 1/3 for oblateness.

The Earth is, to a good approximation, in the shape of an ellipsoid of revolution with the following characteristics:

• oblateness (aplatissement) = 1/298.247; $\dfrac{R_{eq} - R_{po}}{R_{eq}} = \dfrac{1}{298.247}$



Ellipsoïde

• polar radius = 6356.75 km (semi-minor axis of the ellipsoid of revolution);

• equatorial radius = 6378.14 km (semi-major axis of the ellipsoid of revolution);

• gravitational field strength (Presenter)at the surface, at the poles, γ_{pole} = 9.83 m/s²;

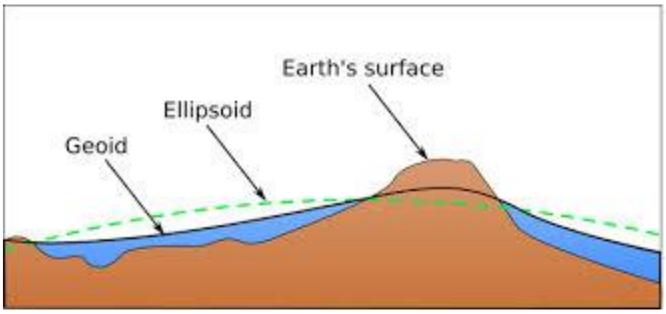
• gravitational field strength at the surface, at the equator, $\gamma_{equator}$ = 9.78 m/s².

The value of gravity varies from **9.83 m/s²** at the North Pole **to 9.78 m/s²** at the equator. Therefore, we are slightly heavier, by 0.5%, at the poles than at the equator.

❑ Earth's "real" shape and GEOID

The geoid is an equipotential surface of the Earth's gravitational field that coincides with the mean sea level at rest and extends beneath the topographic surface of the continents. It is the reference surface for altitudes (level 0). It defines the shape of the Earth.

The reference ellipsoid is an ellipsoid of revolution that most closely approximates the geoid. It corresponds to an equipotential surface of the Earth's theoretical gravitational field.



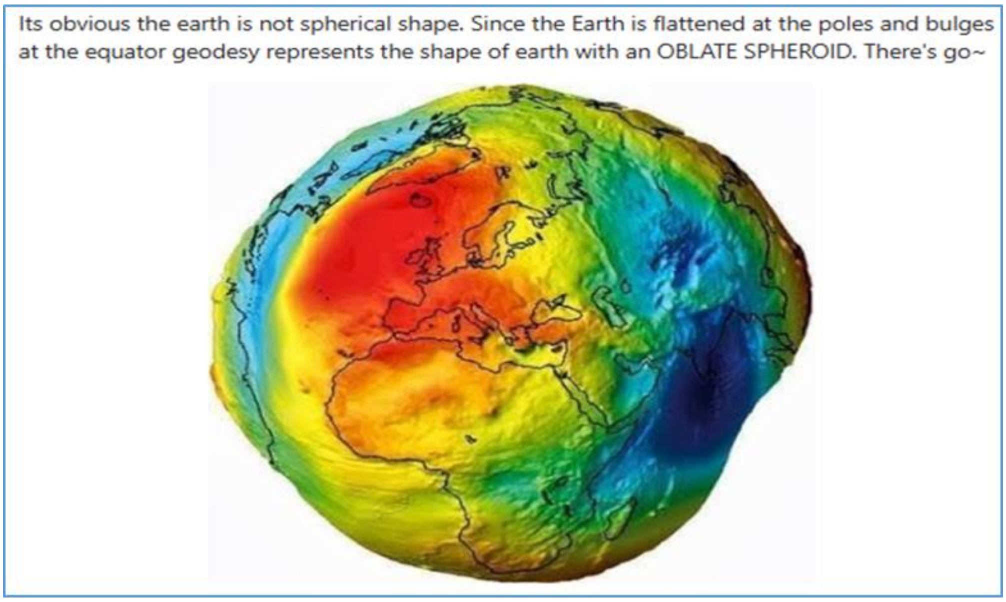
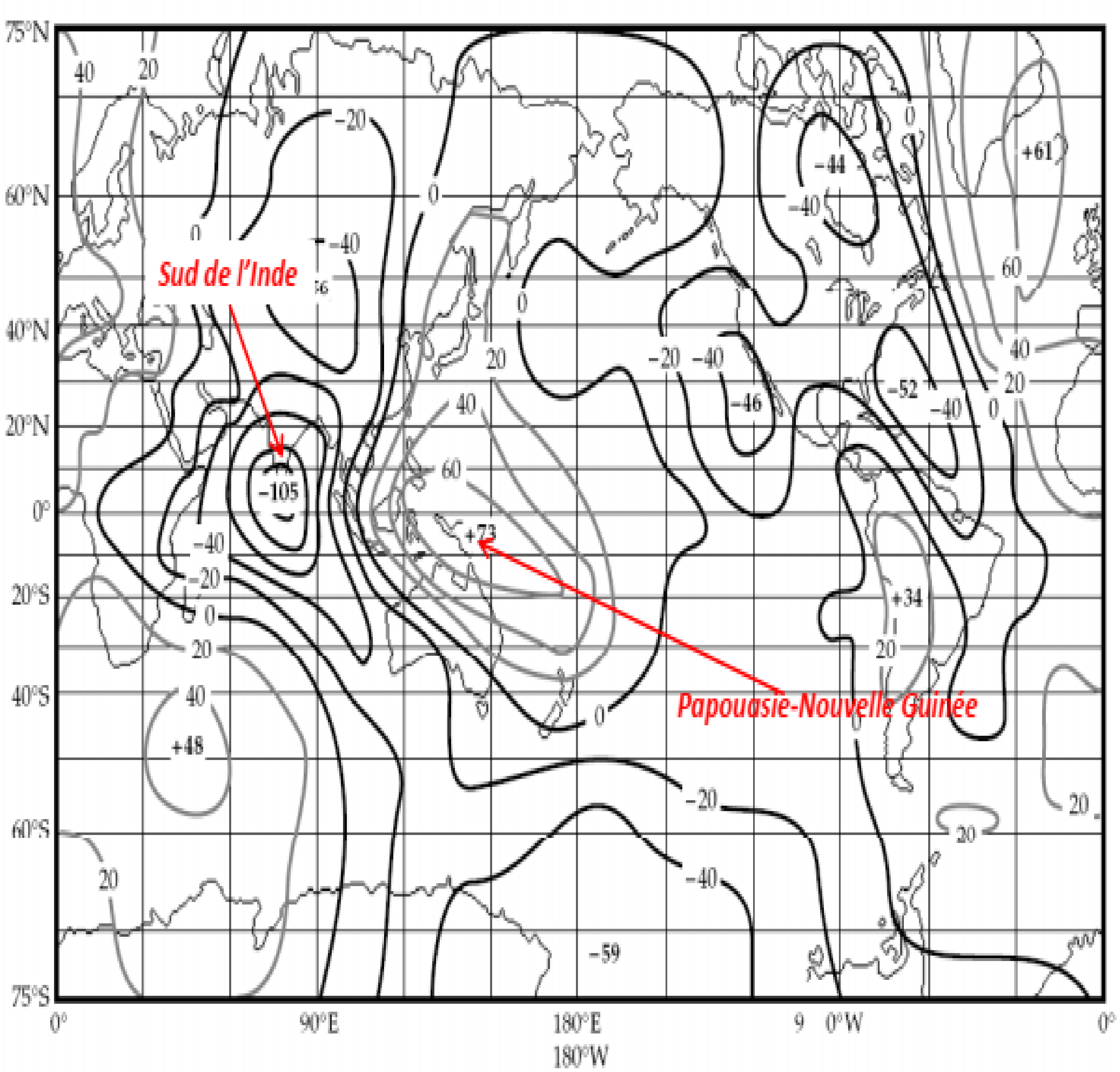


Figure: Actual shape of the earth "Geoid" (CHOUTEAU M., 1999)

The measurement of the gravitational vector, which is perpendicular to the geoid at every point (using the fact that the field lines are perpendicular to the equipotential surfaces). We sometimes speak of the geoid's bumps and hollows... whereas the geoid defines a horizontal reference surface, since at every point it is perpendicular to the vertical line given by gravity. There can only be a bump or hollow in the geoid relative to another shape, a reference ellipsoid.

The geoid and the spheroid do not coincide at every point. Maps exist showing the height of the geoid relative to the spheroid. The two largest variations are in southern India (-105m) and New Guinea (+73m).



World map of geoid undulations relative to a reference ellipsoid of flattening f 1/298.257

II.7 Gravimetric Surveying

Gravimetry is a prospecting method used to determine density anomalies in the subsurface. These minute variations, caused by density contrasts in the Earth's crust constituents, can be measured by gravimetric surveys using highly sensitive instruments. Geophysical surveys are conducted with gravimeters and are always accompanied by a high-precision topographic survey.

II.7.1 Gravity Measurement: The Gravimeter

Absolute gravimeters exist, which measure the total gravitational field g.

Absolute gravimeters are not used for gravimetric prospecting because they are too bulky and their measurement is very complex. Relative gravimeters are used, which measure relative differences in the gravitational field.

A relative gravimeter can be represented schematically by a spring carrying a mass. A small change in attraction, g, will cause a displacement of the mass and also a change in the spring's length x by a small amount x. To measure g with an accuracy of 0.01 mGals, the relative change in the spring's length x/x must be measured with an accuracy of 1 part in 10^8, which is extremely precise.

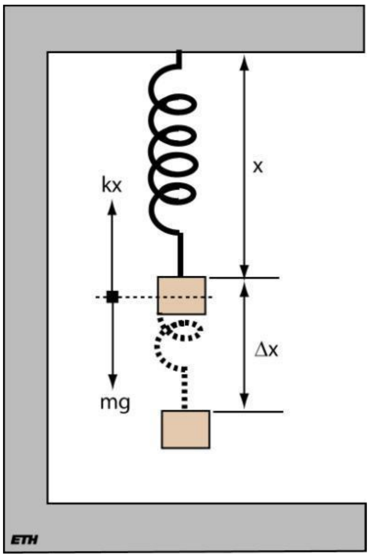


Figure II.5: Diagram of a spring gravimeter. (CHOUTEAU M., 1999)

II.7.2 Gravimetric Data

With a relative gravimeter, relative gravity (between different stations) is measured according to a grid or profile on the ground. It is necessary to know the precise position of each station (accuracy of approximately a few centimeters laterally and better than one centimeter in altitude).

Differential GPS units are often used. Each station is generally measured several times according to a gravimetric measurement cycle (see figure below). These relative points can be linked (the relative measurements are added together) to a gravimetric baseline where the absolute gravity is already known. The measured, or observed, data (gobs) are then obtained.

II.7.2.1 Corrections and References

To obtain variations in the gravitational field due solely to geological causes, the gobs measurements must be corrected for a number of factors in order to define what is called:

Bouguer anomaly (BA): This operation aims to extract from the measured signal any already known information (for example, the surrounding topography) in order to highlight only the anomaly.

This known information is contained in a model.

This g model correction is the gravitational effect of a model composed of a rock layer of imposed density and taking into account the latitude and altitude of the measurement point as well as the surrounding topography. Such a correction is sometimes called "reduction."

Bouguer anomaly

(Pierre Bouguer (1698-1758)

By definition, the Bouguer anomaly will be the difference between the measured value of gravity and

$$AB = g_{mes.} \pm \sum corr. - g_{théor.}$$

g_{mes} *: Is the value of gravity at the measurement point (x, y, z)*

g_{theo} *: Is the value of gravity on the reference ellipsoid at the measurement point (x, y, o)*

$$AB = g_{mes.} - g_{théor.} + 0.3086\, h - 0.0419\, dh + C_T$$

Anomalie de Bouguer simple

the theoretical value after the necessary corrections.

Corrections applied to gravimetric measurements

A. Drift correction

This correction attempts to eliminate the influence of tides (Figure IV.9) and instrument fatigue on the measurements.

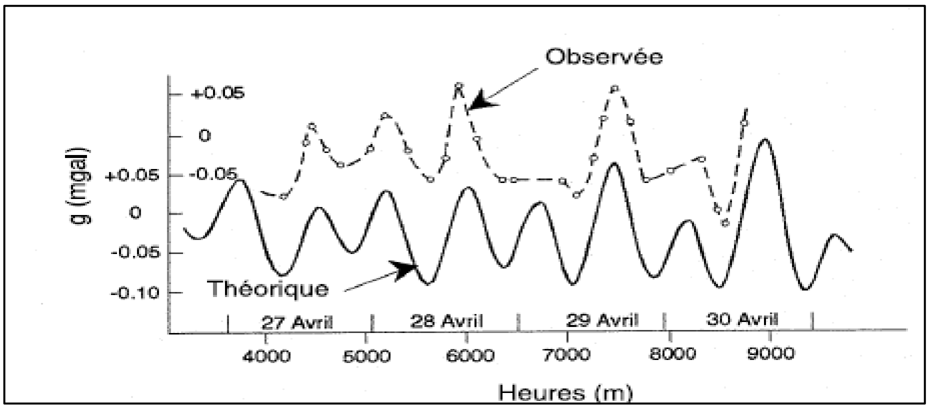


Figure II.10: Lunar tide

To this end, it is necessary to follow a specific path between the reading stations.

In practice, a series of measurements is taken following a loop path: the series usually begins at a given point and ends at the same point. The starting point of the loop is normally connected to a base station.

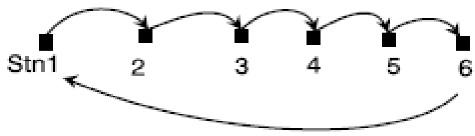


Figure II.11: Loop measurement method

In general, the measurements taken at the start and end of the journey at the base station are not the same. This difference, called drift, is due partly to the gravimeter and partly to lunar tides.

The correction is made assuming that the drift is linear over time. Therefore, if we have passed through the base station at times T1 and T2 and the measured values were V1 and V2 respectively, the drift rate TD is defined by:

$$TD = \frac{V_2 - V_1}{T_2 - T_1}$$

When the drift is positive, it means the measurements have been overestimated and must therefore be reduced. The drift correction will be negative. Conversely, if the drift is negative, the measurements are underestimated and the correction must be positive.

Thus, any value V taken at time T (where $T_1 \leq T \leq T_2)$) is corrected by the following formula.

$$V_{cor} = V_{lu} - \left[\frac{V_2 - V_1}{T_2 - T_1}\right](T - T_1)$$

B. Latitude Correction

This correction takes into account the variations of g with latitude due to the Earth's rotation and its oblateness.

From worldwide geodetic measurements, we know that the Earth is an almost perfect ellipsoid of revolution. On this surface, the gravitational field can be described by the following equation (I.U.G.G. 1967):

$$g_{th}(\varphi) = 978.03[1 + 5.2789x10^{-3}sin^2\varphi + 23.642x10^{-6}sin^4\varphi]gals$$

where $g_{th}(\varphi)$ is the value of the field at the geocentric latitude point φ. The Δ_L correction for a displacement dl along a meridian is therefore:

$$\Delta_L = \frac{dg_{th}}{dl}.dl\ avec\ dl = R(\varphi)d\varphi \approx R_e d\varphi$$

Where $\mathbf{R_e}$ is the Earth's equatorial radius (6378 km)? Finally,

$$\Delta_L = 0.081dlsin2\varphi\frac{mgal}{100m}: (N \longrightarrow S)$$

Calculation example

This correction takes into account the variations of g with latitude due to the Earth's rotation and its oblateness. The correction is positive towards the north and negative towards the south, as for example here with an origin point at coordinates (48°44'N):

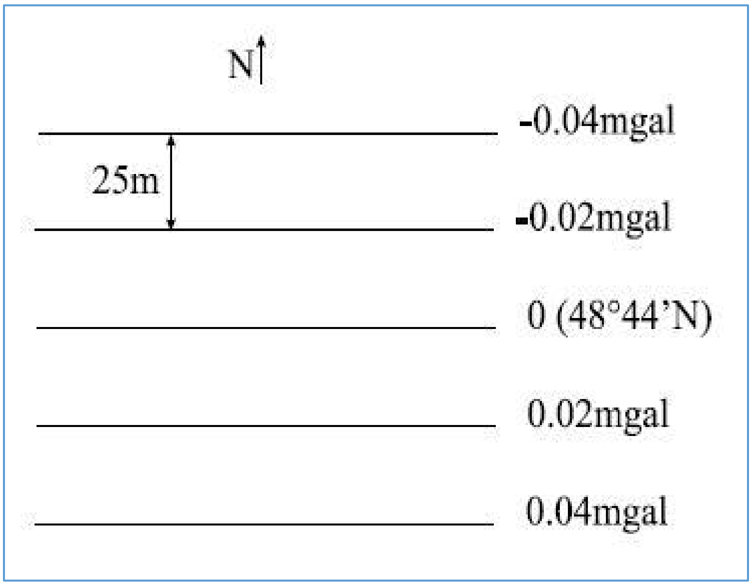


Figure II.13: Example of latitude correction for a reference point with coordinates 48°44'N (CHOUTEAU M., 2002)

1. Altitude correction (free air, Faye) (FAC)

Gravimetric survey readings are not necessarily taken over flat terrain.

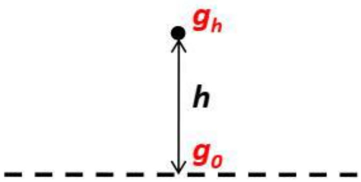
However, the closer one gets to the reference level, the greater the g-force. The measurements obtained therefore exhibit variations that are solely due to the position of the measurement station and not to heterogeneities in the subsoil. The measurements must therefore be corrected.

Since $g_r = \frac{Gm}{r^2}$

Where $\boldsymbol{R}$ is the radius of the Earth at the reference level, if we move a height $\boldsymbol{h}$ relative to this reference level, then

$$g_0 = \frac{GM}{R^2}$$

$$g_h = \frac{GM}{(R+h)^2} = \frac{GM}{R^2 + 2Rh + h^2} = \frac{GM}{R^2\left(1 + 2\frac{h}{R} + \left(\frac{h}{R}\right)^2\right)}$$



Puisque R>>h, alors le terme (h/R)² peut être négligé

$$g_h = \frac{GM}{R^2}\,\frac{1}{1+2\frac{h}{R}}\,\frac{1-2\frac{h}{R}}{1-2\frac{h}{R}} = \frac{GM}{R^2}\left(1 - 2\frac{h}{R}\right) \qquad g_h = g_0 - 2g_0\frac{h}{R}$$

$\Rightarrow$ $$\Delta g = g_h - g_0 = -2g_0\frac{h}{R}$$ ***Représente le gradient vertical de g à l'air libre***

- From this expression, we deduce that an increase in altitude of 1 m would result in a decrease in gravity of 0.3086 mGal.
- For a measurement accuracy of 0.01 mGal, the altitude would need to be known to within ± 3.3 cm.

Ce gradient vertical varie :

de 0.3077 mGal/m $\Rightarrow$ ***À l'équateur (g= 978.0 Gal et R = 6378 km)***

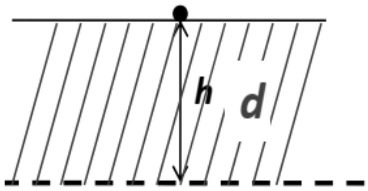
à 0.3093 mGal/m $\Rightarrow$ ***Aux pôles (g= 983.2 Gal et R = 6357 km)***

Tenant compte de la forme de la terre, la valeur normale acceptée pour la <u>correction à l'air libre</u> est de :

$$\Delta g = -0.3086\, h$$

A. Plateau Correction

This correction is also related to the altitude of the measurement point. Since there is, in fact, a layer of terrain with density "d" beneath the measurement point, the gravitational acceleration will be increased by its gravitational pull. Therefore, the effect of the mass between the reference level and the measurement point must be taken into account.

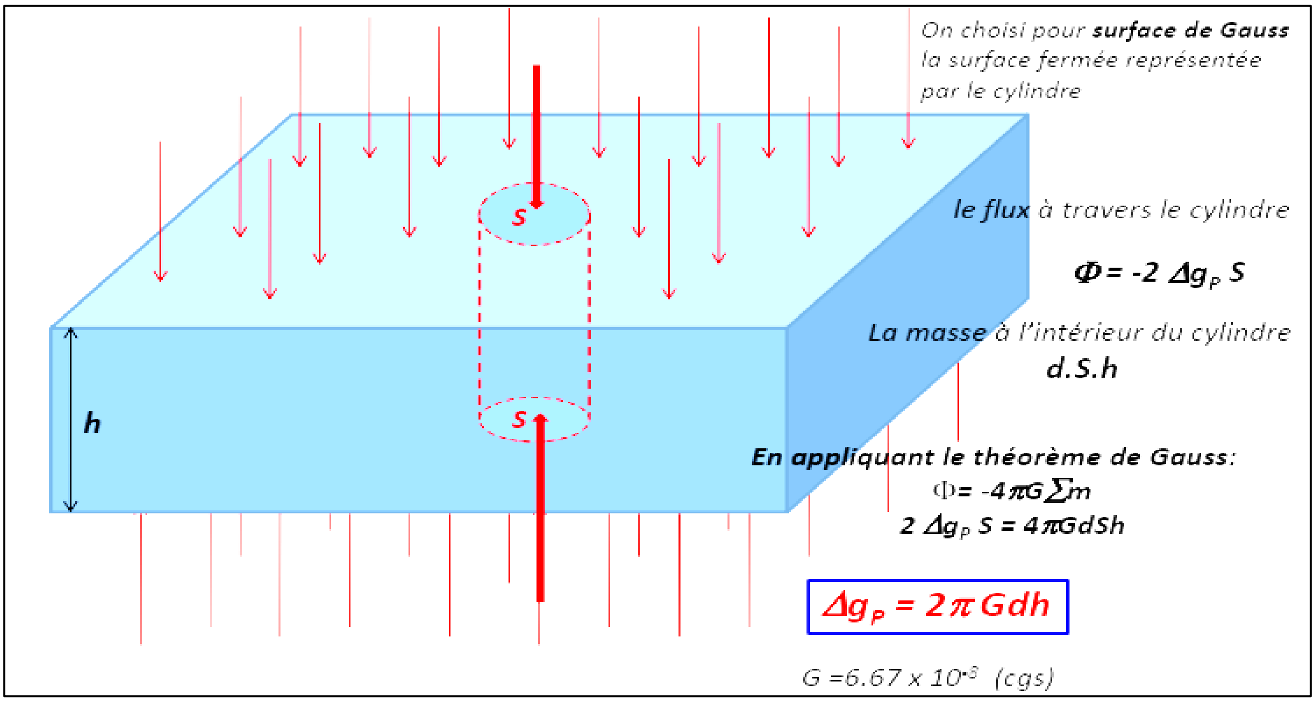


For such a layer of height h, its gravimetric effect can be obtained using Gauss's law.

The mass of this layer of terrain is approximated by a horizontal and homogeneous layer of unlimited extent.

Since the plateau effect increases g, we would therefore need to subtract Δg_P. Therefore, this correction is negative and equals:

$$\Delta g_P = -0.0419dh \quad \text{en mGal} \quad (h>0)$$



Another way to simplify:

The plateau correction takes into account the mass between the reference frame and the measurement station.

For a height segment h, the attraction is given by: $\Delta g_P = 2\pi G \rho_B h$

Where **G** = universal gravitational constant and ρ_B is the assumed density of the Earth's crust ($\rho_B = 2.67\ g/cm^3$ (on average).

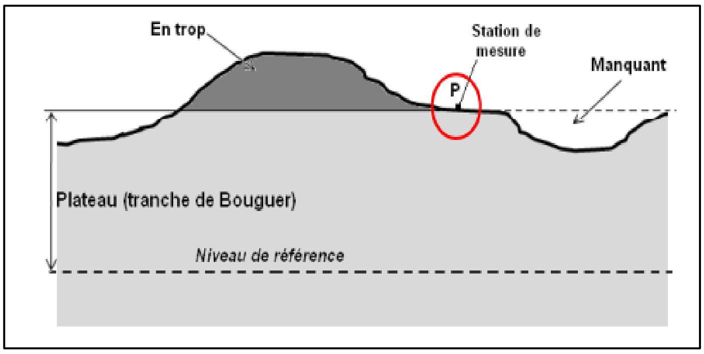
Since Δg_P increases when ***h*** increases, we must subtract Δg_P when h > 0, and therefore:

$$\Delta g_P = -0.04191\, \rho_B h \; \frac{mgal}{m}; (h > 0)$$

B. Relief Correction (Topographic)

For the Bouguer correction, we considered a horizontal terrain slice with a thickness equal to the altitude of the measurement point. In reality, the ground surface is not flat; therefore, it would be necessary to numerically integrate the portions that extend beyond and, on the other hand, the portions that are missing from the Bouguer slice.

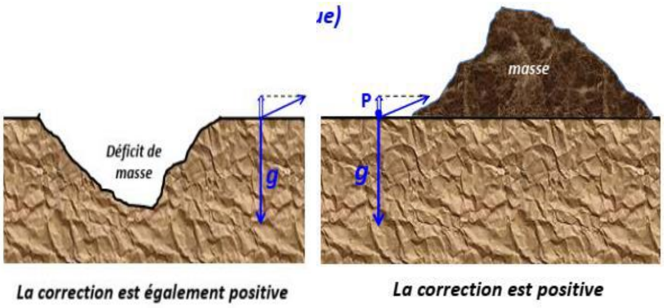
- ❖ It would be necessary to compensate for the effects of the terrain's "bumps" and "dips."



- ❖ Terrain corrections always have the same sign.

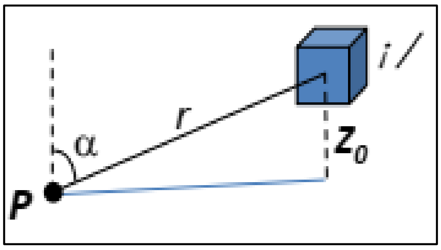
- ❖ Indeed, both a "bump" and a "dip" tend to decrease the value of g.

في الواقع، يميل كلٌّ من "النتوء" و"الانخفاض" إلى خفض قيمة g.



The medium with density **Δρ** exerts an attraction on point **P** whose magnitude of the vertical component is:

$$\Delta g_t = G\Delta\rho \int_V \frac{dv}{r^2} \cos\alpha$$



r being the distance between a volume element **dV** and the point **P**. **α** is the angle between r and the vertical at the point P.

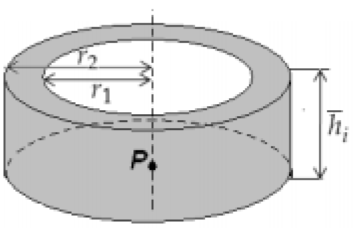
$$\Delta g_t = G\Delta\rho \int_V \frac{z_o}{(x_o^2 + y_o^2 + z_o^2)^{3/2}} dv.$$

For extra pieces, $\mathbf{Z_o}$ and $\mathbf{\Delta\rho}$ are positive, and for missing pieces, $\mathbf{Z_o}$ and $\mathbf{\Delta\rho}$ are negative. Thus, the relief correction is always positive.

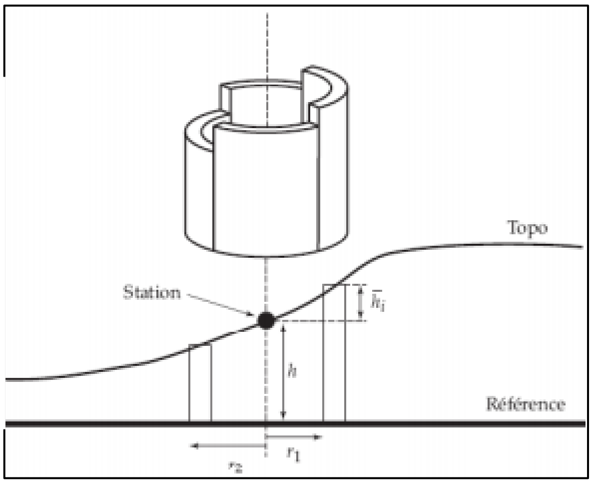
Generally, the integration is performed digitally using a computer and digitized topographic maps.

- To simplify calculations, the terrain can be discretized into concentric rings, which are themselves divided into segments whose vertices are fitted to the average topography.

- The expression giving the gravitational effect on the axis of a ring of thickness r2 -r1 is as follows.

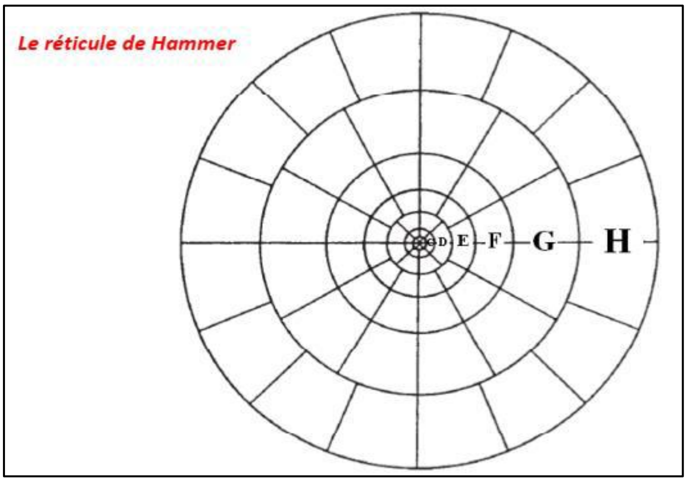


$$\Delta g_c = 2\pi G\rho \left[r_2 - r_1 + \sqrt{r_1^2 - \overline{h}_i^2} - \sqrt{r_2^2 - \overline{h}_i^2} \right]$$



Given the symmetry of the problem, the effect of a single section of the crown is equal to the overall effect of the entire crown divided by the number of sections by which the crown was divided.

$$\Delta g_i = \frac{2\pi G\rho}{N} \left[r_2 - r_1 + \sqrt{r_1^2 - \overline{h}_i^2} - \sqrt{r_2^2 - \overline{h}_i^2} \right]$$



In practice, we use a reticle which we superimpose on the topographic maps, then we refer to the tables prepared by **Hammer** (and completed by Bible) which give us, for different values of **h**, the corrections in **mgal** that we need to make for each of the sectors of the reticle.

Tables de Hammer pour la correction de relief ; densité : r = 2 g/cm³ ; Zones B à M; Correction : $\sum t_i$ * 0.01 mGal

Zone	B	C	D	E	F	G	H	I	J	K	L	M
Secteurs	4	6	6	8	8	12	12	12	16	16	16	16
Rayon	2 à 16.5 m	16.6 à 53.3 m	53.3 à 170.1 m	170.1 à 390 m	390 à 895 m	895 à 1529 m	1529 à 2615 m	2615 à 4470 m	4470 à 6650 m	6650 à 9900 m	9900 à 14750 m	14750 à 21950 m
t_i	±h en m	±h en m	±h en m	±h en m	±h en m	±h en m	±h en m	±h en m	±h en m	±h en m	±h en m	±h en m
0	0.0 à 0.3	0.0 à 1.3	0.0 à 2.4	0.0 à 5.5	0.0 à 8.2	0.0 à 17.6	0.0 à 22.9	0 à 30.2	0 à 51	0 à 62	0 à 76	0 à 93
0.1	0.3 à 0.6	1.3 à 2.3	2.4 à 4.1	5.5 à 9.1	8.2 à 14.0	17.6 à 30.5	22.9 à 40.0	30.2 à 52.1	51 à 88	62 à 108	76 à 131	93 à 160
0.2	0.6 à 0.8	2.3 à 3.0	4.1 à 5.3	9.1 à 12.0	14.0 à 18.3	30.5 à 39.3	40.0 à 51.5	52.1 à 67.1	88 à 114	108 à 139	131 à 170	160 à 207
0.3	0.8 à 0.9	3.0 à 3.5	5.3 à 6.3	12.0 à 14.3	18.3 à 21.6	39.3 à 46.6	51.5 à 61.0	67.1 à 79.6	114 à 135	139 à 165	170 à 201	207 à 245
0.4	0.9 à 1.0	3.5 à 4.0	6.3 à 7.1	14.3 à 16.2	21.6 à 24.4	46.6 à 52.7	61.0 à 68.9	79.6 à 90.2	135 à 153	165 à 187	201 à 228	245 à 278
0.5	1.0 à 1.1	4.0 à 4.4	7.1 à 7.8	16.2 à 17.7	24.4 à 27.0	52.7 à 58.0	68.9 à 76.0	90.2 à 100	153 à 169	187 à 206	228 à 252	278 à 307
1	1.1 à 2.0	4.4 à 7.3	7.8 à 13.1	17.7 à 29.6	27.0 à 45.0	58.0 à 97.0	76.0 à 126	100 à 164	169 à 280	206 à 341	252 à 416	307 à 507
2	2.0 à 2.7	7.3 à 9.8	13.1 à 17.0	29.6 à 38.3	45.0 à 58.0	97.0 à 125.	126. à 163	164 à 213	280 à 361	341 à 441	416 à 537	507 à 655
3	2.7 à 3.5	9.8 à 11.9	17.0 à 20.2	38.3 à 45.4	58.0 à 68.0	125 à 148	163 à 193	213 à 253	361 à 427	441 à 521	537 à 636	655 à 776
4	3.5 à 4.2	11.9 à 13.8	20.2 à 23.1	45.4 à 51.8	68.0 à 77.0	148 à 158	193 à 219	253 à 287	427 à 485	521 à 591	636 à 721	776 à 880
5	4.2 à 5.0	13.8 à 15.6	23.1 à 25.7	51.8 à 57.6	77.0 à 86.0	158 à 186	219 à 242	287 à 317	485 à 537	591 à 654	721 à 797	880 à 978
6	5.0 à 5.7	15.6 à 17.4	25.7 à 28.1	57.6 à 62.9	86.0 à 94.0	186 à 202	242 à 263	317 à 344	537 à 584	654 à 711	797 à 867	973 à 1058
7	5.7 à 6.5	17.4 à 19.1	28.1 à 30.4	62.9 à 67.8	94.0 à 101	202 à 213	263 à 283	344 à 369	584 à 628	711 à 764	867 à 932	1058 à 1136
8	6.5 à 7.3	19.1 à 20.8	30.4 à 32.6	67.8 à 72.4	101 à 108	213 à 233	283 à 302	369 à 393	628 à 669	764 à 814	932 à 993	1136 à 1210
9	7.3 à 8.2	20.8 à 22.6	32.6 à 34.7	72.4 à 76.8	108 à 114	233 à 247	302 à 320	393 à 416	669 à 708	814 à 861	993 à 1050	1210 à 1280
10	8.2 à 9.1	22.6 à 24.4	34.7 à 36.7	76.8 à 81.1	114 à 120	247 à 260	320 à 337	416 à 438	708 à 745	861 à 906	1050 à 1104	1280 à 1346
11	...	24.4 à 26.1	36.7 à 38.7	81.1 à 85.3	120 à 126	260 à 272	337 à 353	438 à 459	745 à 780	...	...	...
12	...	26.1 à 27.9	38.7 à 40.6	85.3 à 89.3	126 à 131	272 à 284	353 à 368	459 à 479	780 à 813	...	...	...
13	...	27.9 à 29.7	40.6 à 42.6	89.3 à 93.2	131 à 137	284 à 296	368 à 383	479 à 498	813 à 845	...	...	...
14	...	29.7 à 31.6	42.6 à 44.5	93.2 à 97.0	137 à 142	296 à 308	383 à 397	498 à 516	845 à 877	...	...	...
15	...	31.6 à 33.5	44.5 à 46.4	97.0 à 100.8	142 à 147	308 à 319	397 à 411	516 à 534	877 à 908	...	...	...

Tableau 2.1– **Différentes densités des matériaux terrestres.**

matériaux	densité	matériaux	densité
Densité moyenne de la Terre	5,5	Gabbros	2,7 à 3,3
Densité moyenne de la croûte continentale	2,67	Péridotite	3,1 à 3,4
Sédiments non consolidés	1,8 à 2,0	Charbon	1,2 à 1,8
Sables « secs »	1,4 à 1,65	Pétrole	0,6 à 0,9
Sables « humides »	1,9 à 2,05	Eau de mer	1,01 à 1,05
Grès	2,0 à 2,5	Glace	0,88 à 0,92
Sel	2,1 à 2,4	Chromite	4,5 à 4,8
Marnes	2,1 à 2,6	Pyrite	4,9 à 5,2
Calcaires	2,4 à 2,8	Hématite	5,0 à 5,2
Granites	2,5 à 2,7	Magnétite	5,1 à 5,3
Dolérite	2,5 à 3,1	Fer	7,3 à 7,8
Serpentine	2,5 à 2,6	Cuivre	8,8 à 8,9
Gneiss	2,65 à 2,75	Argent	10,1 à 11,1
Basaltes	2,7 à 3,1	Or	15,6 à 19,4

II.7.3 Bouguer Anomaly

The Bouguer anomaly is: $\Delta g_B = \Delta g$ **(observed)** $\mp$ **the** $5\ corrections$

1- Aircraft drift correction

2- Latitude correction $\Delta_L = 0.081 sin^2\varphi$ mgal/100m

3- Altitude correction $\Delta_h = 0.3086\, h$ mgal/m

4- Plateau correction $\Delta_B = -0.04191 \rho_B h$ mgal/m

5- Terrain correction Δ_T

Note:

The gravimeter does not give an absolute value of g, but rather a relative value.

$$\Delta g_B = g_{observée} \mp \Delta g - g_{ref}$$

Thanks to the corrections, the observed value in x, y, and z has been made comparable to the value calculated on the reference ellipsoid at (x; y; z) = 0. We therefore conclude that the observed value has been reduced to the level of the ellipsoid.

II.7.4 Isostasy: Hydrostatic Behavior of the Lithosphere

The difference between the calculated theoretical gravity and the measured and corrected gravity can be significant,

on the order of -300 to +300 mgal. It is negative over mountain ranges and positive over oceans. It is as if the Bouguer reduction calculation were largely unnecessary and that the excess mass created by the mountainous relief above the geoid was already almost compensated (before any correction) below the chosen reference surface with the correction introducing an apparent mass “deficit.” This anomaly <0 therefore largely results from the correction calculation itself, and the weight of a column of terrain ultimately appears constant from one vertical to the other. Thus, from one vertical to the other, a hydrostatic compensation, called isostasy, takes place.

Several models have been developed (Figure IV.8). Everyone agrees that the crust, being rigid and less dense than the fluid mantle, floats on it (Archimedes). The equilibrium of the weights of the different columns of rock is therefore perfectly achieved beyond a certain depth.

called compensation. **Pratt's model**, at the end of the last century, involved columns of soil with different densities, and therefore different thicknesses. **Airy's** model, a few years later, involved columns of the same density discretizing a homogeneous layer. Finally, **Vening Meinesz's model**, dating from the mid-20th century, is based on the fact that **the elastic response of the crust** is of a much greater amplitude than the size of the landforms it supports. In other words, the isostatic compensation of a landform is regional.

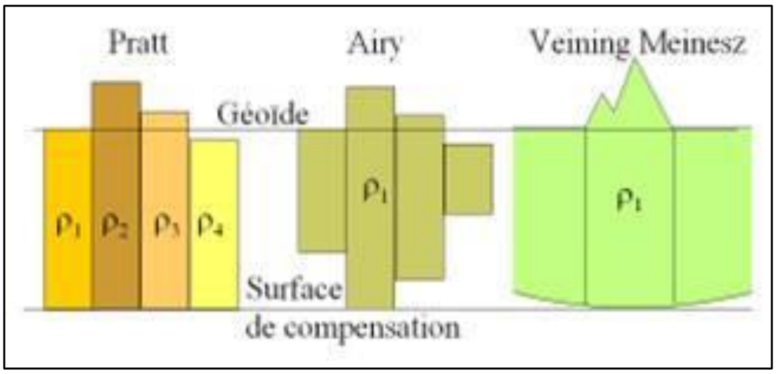


Figure II.8: Isostasy models

The influence of the roots can be expressed by a new correction, called "isostatic."

However, the existence of residual anomalies shows that equilibrium is not achieved everywhere.

This is due to the very high viscosity of the mantle, which implies a significant response time to any change in equilibrium. Thus, the Baltic Shield is still rising today, even though it has been freed from the weight of the ice that covered it for approximately 10,000 years.

The first essential lesson of gravimetry is therefore the confirmation of the Earth's plasticity, thanks to its viscoelastic mantle, despite its elastic (rigid) crust.

The equipotential surface of gravity coinciding with mean sea level, extended across the continents, is called the geoid. If the Earth were ellipso-concentric in all its properties, the geoid would be an ellipsoid, and the isotherms would be ellipsoids "conforming" to the geoid.

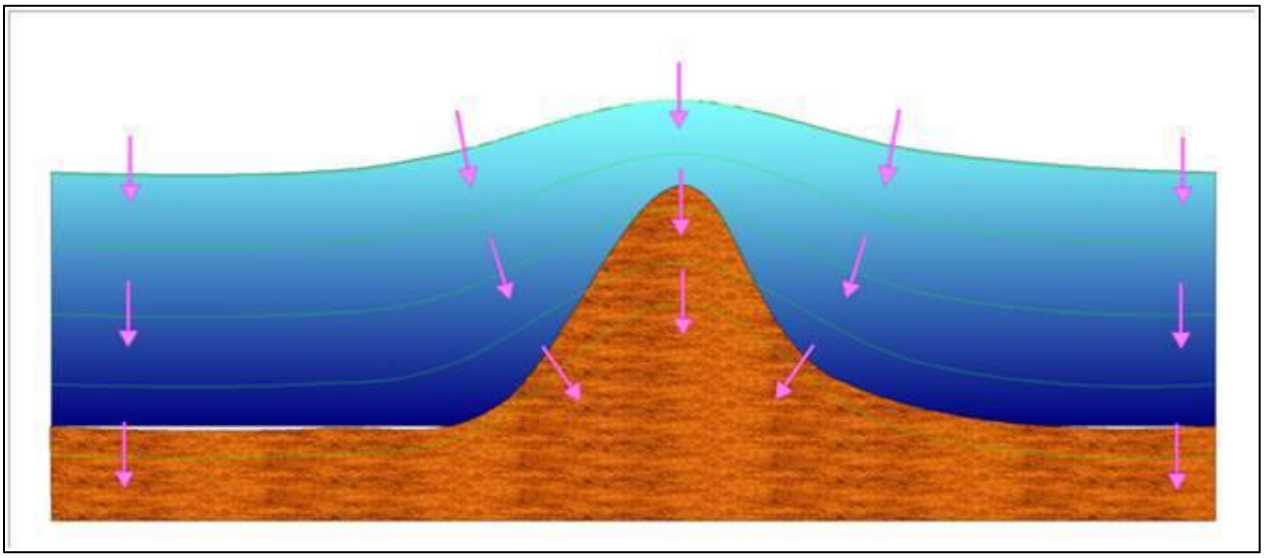
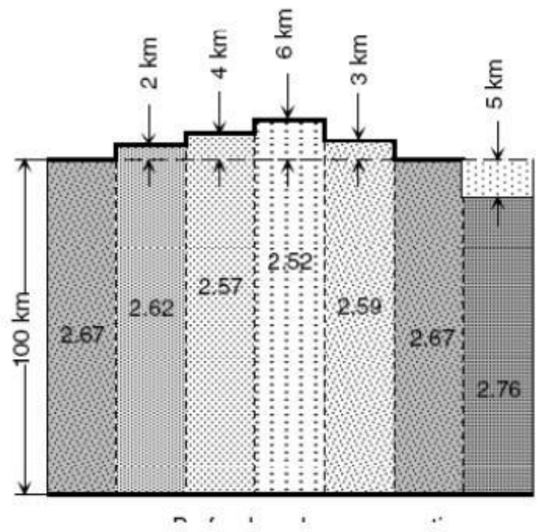


Figure II: Effect of underwater topography on gravity deflection and sea level (geoid).

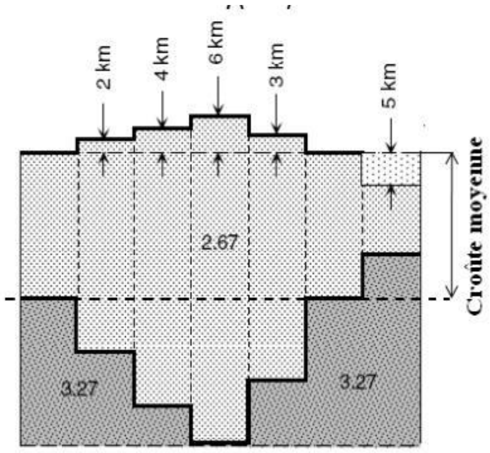
Systematic Nature of Bouguer Anomalies:

Examination of large-scale Bouguer anomaly maps shows that they are:

- ✓ weak in absolute value over low-lying plains
- ✓ positive over oceans
- ✓ negative over mountains (increasing in absolute value with altitude). - There are two models:



$$d_{plaine} \times h_0 = p_{océan} \times 1 + d_{océan}(h_0 - p) = d_{montagne}(h_0 + h_m)$$



II.8 Interpretation of Gravimetric Data

The most important problem in gravimetry applied to prospecting is separating anomalies due to different causes, in order to study and interpret them individually. This separation can be described as preliminary or qualitative interpretation; in fact, it is, in most cases, sufficient for prospecting needs. The quantitative study and interpretation of isolated anomalies constitute the second gravimetric problem.

The goal of interpreting anomalies is to find the distribution of sources:

- Density contrasts
- Geometries that create the anomalies observed at the surface.

It can be shown theoretically that gravimetric data alone are not sufficient to uniquely determine a mass distribution at depth; several very different geometries can create the same gravimetric anomalies. In the figure below, each of the three bodies represented creates the same effect at the surface.

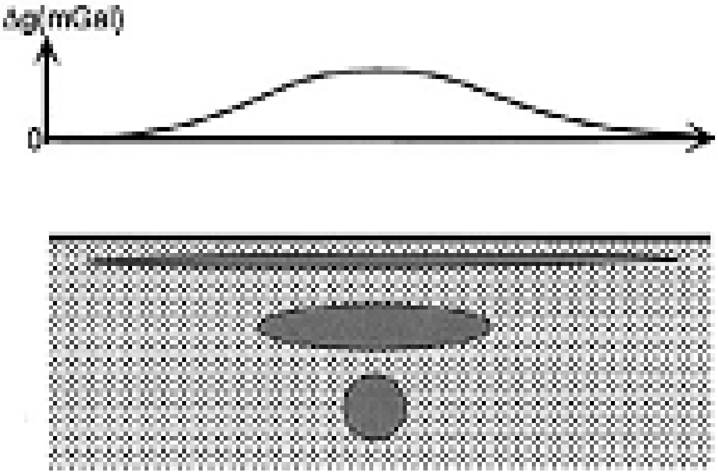


Figure II.14: Anomalies de « grande longueur d'onde » générée soit par une source ponctuelle profonde soit par une source superficielle.

II.8.1 Effects of Simple Structures

The gravimetric effects created by simple geometric structures can be obtained analytically through direct calculation. Here, we will focus on the vertical component, the one that can be measured with gravimeters.

a) Case of a Sphere

The vertical component of the attraction created by a sphere of mass M, assumed to be concentrated at its center located at a depth **h** – this therefore corresponds to the effect of a point source of mass **M** at depth **h** – on a point **x** on a horizontal axis passing vertically through the center of the sphere is:

$$g(x) = \frac{GMh}{(x^2 + h^2)^{3/2}}$$

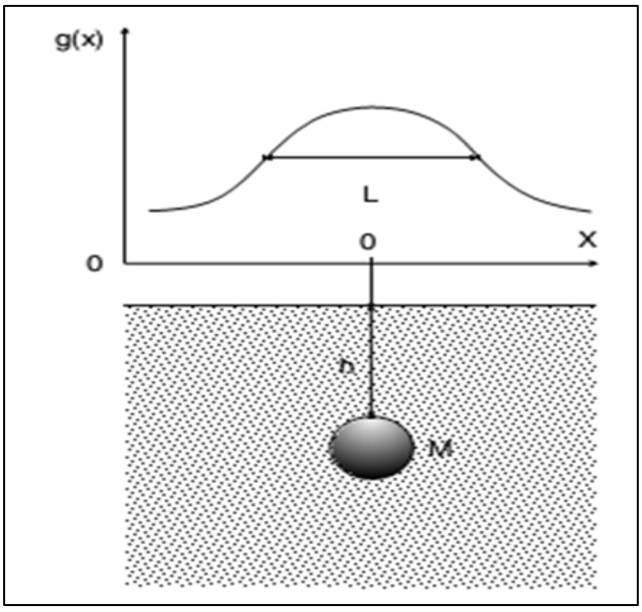


Figure II.15: Gravimetric effect created by a spherical source M whose mass is concentrated at its center located at a depth h.

We can see, therefore, that the maximum of this effect, at x=0, is proportional to the inverse square of the source depth h, and that the full width at half maximum (FWHM) of the signal is proportional to h.

The FWHM is defined by L = 2x, with x such that g(x) = 0.5gmax. That is:

$$\frac{GMh}{(x^2+h^2)^{3/2}} = 0.5\frac{GM}{h^2}, donc\ L = 1.53h, ou\ h = 0.65L$$

Note:

1. The deeper the source, the lower the amplitude of the associated signal and the longer its wavelength.

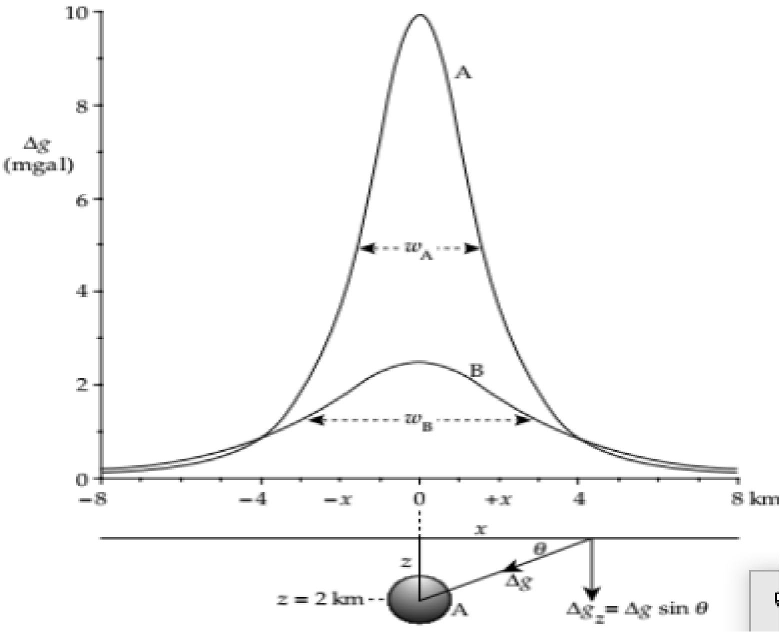
To better understand

$$\Delta g_z = \Delta g \sin\theta = G\frac{M}{r^2}\frac{z}{r}$$

$$M = \frac{4}{3}\pi R^3 \Delta\rho \quad \text{and} \quad r^2 = z^2 + x^2$$

$$\Delta g_z = \frac{4}{3}\pi G \Delta\rho R^3 \frac{z}{(z^2+x^2)^{3/2}}$$

$$= \frac{4}{3}\pi G\left(\frac{\Delta\rho R^3}{z^2}\right)\left[\frac{1}{(1+(x/z)^2)}\right.$$



The maximum anomaly is found at x = 0 and is equal to:

$$\Delta g_0 = \frac{4}{3}\pi G\left(\frac{\Delta\rho R^3}{z^2}\right)$$

The distance between the inflection points of the anomaly = the depth of the center of the sphere.

Let us now examine the particular point on the curve where: Δg(x1/2) = Δgmax/2 (see figure) defining the point x = x1/2 (x1/2 is called the "**half-width**" at the "**half-height**"). We have, by setting:

$$V = \frac{4}{3}\pi a^3,$$

$$\Delta g(x_{1/2}) = \Delta g_{max}/2 \Rightarrow \quad VG\Delta\rho \frac{z}{(x_{1/2}^2 + z^2)^{3/2}} = VG\Delta\rho \frac{1}{2z^2}$$

$$\frac{z^3}{(x_{1/2}^2 + z^2)^{3/2}} = \frac{1}{2}$$

$$\left[\frac{z}{(x_{1/2}^2 + z^2)^{1/2}}\right]^3 = \frac{1}{2}$$

$$\left[\frac{1}{\left(1 + \frac{x_{1/2}^2}{z^2}\right)^{1/2}}\right]^3 = \frac{1}{2}$$

$$\left[1 + \left(\frac{x_{1/2}}{z}\right)^2\right]^{3/2} = 2$$

$$\left[1 + \left(\frac{x_{1/2}}{z}\right)^2\right]^3 = 4$$

d'où on trouve finalement

$$\boxed{z = 1.306\, x_{1/2}}$$

It is therefore possible to determine the depth z of the sphere's center from the gravimetric anomaly it produces. Once z is known, the sphere's excess mass (ΔM) can then be calculated.

$$\Delta M = \frac{\Delta g_{max} \cdot z^2}{G}$$

and, if the densities of the surrounding medium and of the sphere are known, the actual mass of the sphere (M) is then obtained by the rule of three

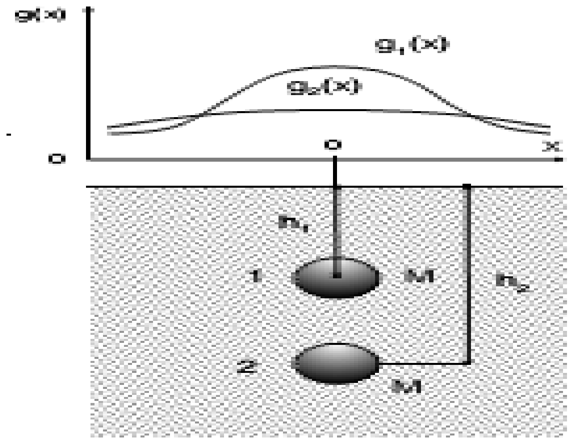
$$M = \Delta M \frac{\rho_{\text{sphère}}}{\Delta\rho}$$

$$\text{où} \quad \Delta\rho = \rho_{\text{sphère}} - \rho_{\text{encaissant}}$$

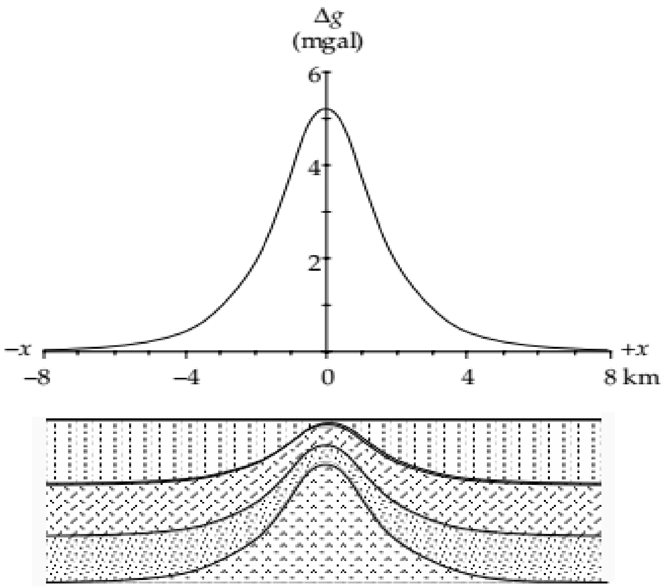
Special case:

Two point sources of the same mass, but located at different depths, produce different effects. This different signal shape depending on depth must be taken into account when preparing surveys. The interval between measurements must be chosen based on the maximum depth of the sources that one seeks to detect

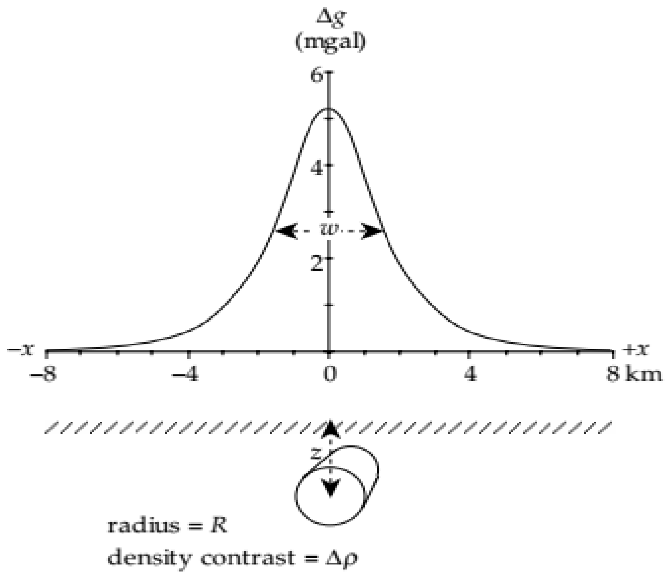
.



b) Case of a cylinder



(a) Anticlinal structure in cross-section



(b) Model as an infinite horizontal cylinder

$$\Delta g_z = \frac{2\pi G R^2 \Delta\rho}{z(1+(x/z)^2)}$$

$$\Delta g_z = \frac{2Gmz}{z^2 + x^2}$$

with a maximum at x = 0 gives by $\Delta g_{max} = \dfrac{2\pi G R^2 \Delta\rho}{z}$.

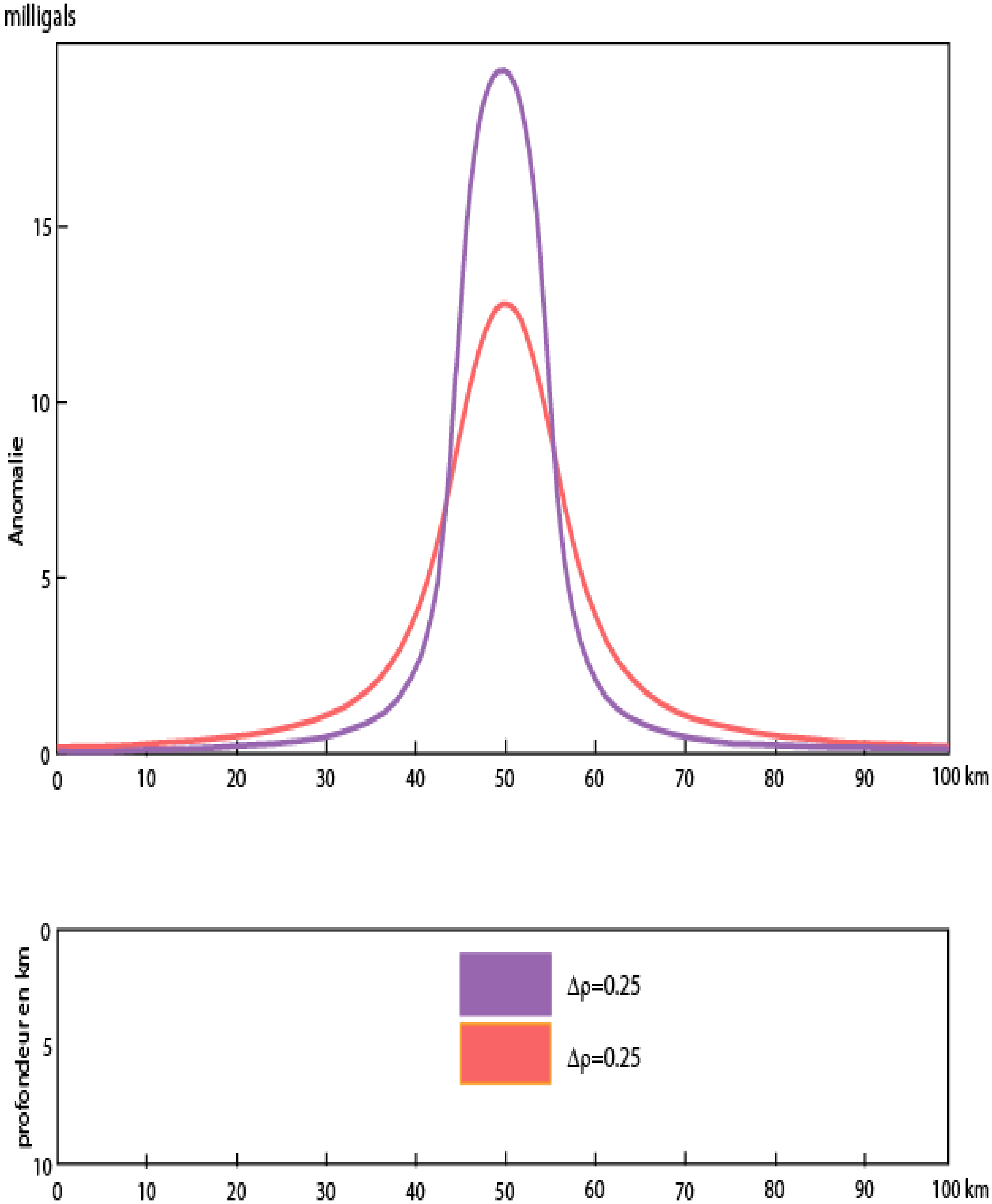
Using these results, we have, at the point x = x1/2, by setting C = 2πGR2

$$
\begin{aligned}
\Delta g &= \frac{\Delta g_{max}}{2} \\
\frac{C\Delta\rho}{z(1+(x_{1/2}/z)^2)} &= \frac{1}{2}\frac{C\Delta\rho}{z} \\
\frac{1}{(1+\left[\frac{x_{1/2}}{z}\right]^2)} &= \frac{1}{2} \\
(1+\left[\frac{x_{1/2}}{z}\right]^2) &= 2
\end{aligned}
$$

$$\boxed{z = x_{1/2}}$$

The depth of the cylinder is found directly using the value of x1/2. Furthermore, the cylinder produces a larger anomaly than a sphere.

Effect of depth on the gravimetric anomaly



The anomaly widens and decreases in amplitude with depth.

II.8.3 Regional Anomaly and Separation of Sources

The anomalies obtained at the Earth's surface will therefore reflect all the heterogeneities of masses at depth. Deep anomalous bodies will generate anomalies of long wavelengths, while surface masses will be the source of anomalies of short wavelengths. However, while deep sources cannot generate anomalies of short wavelengths, the reverse is not true. Indeed, a very large surface source can also generate anomalies of long wavelengths.

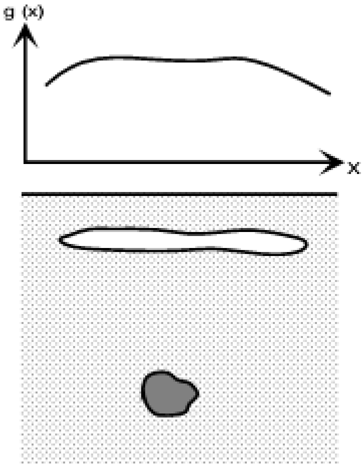


Figure IV.19: "Long wavelength" anomalies generated either by a deep point source or by a shallow source. This indicates an excess of mass in the subsoil.

II.8.3.1 Separation of Anomalies

The Bouguer anomaly obtained is corrected for the effects taken into account in the model. A residual anomaly is then extracted, that is, a local deviation of the regional gravitational field (or regional anomaly obtained by smoothing a surface with a large radius of curvature) (see figure below). The residual anomaly is therefore assumed to reflect the presence of the target we are trying to locate.

The total signal is shown at the top.

The regional anomaly is in the middle,

and the residual signal is at the bottom.

($g$total = $g$regional +$g$residual)

- (g resid = g tot – g reg)

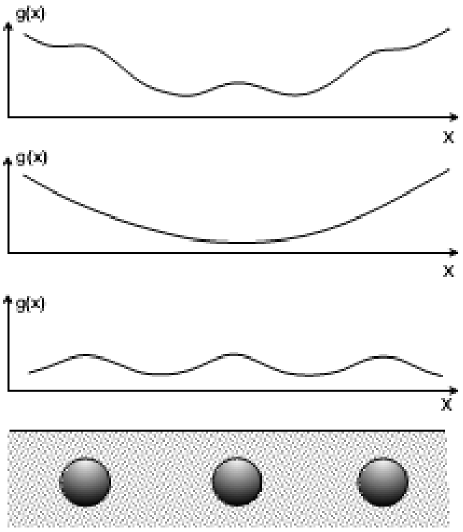
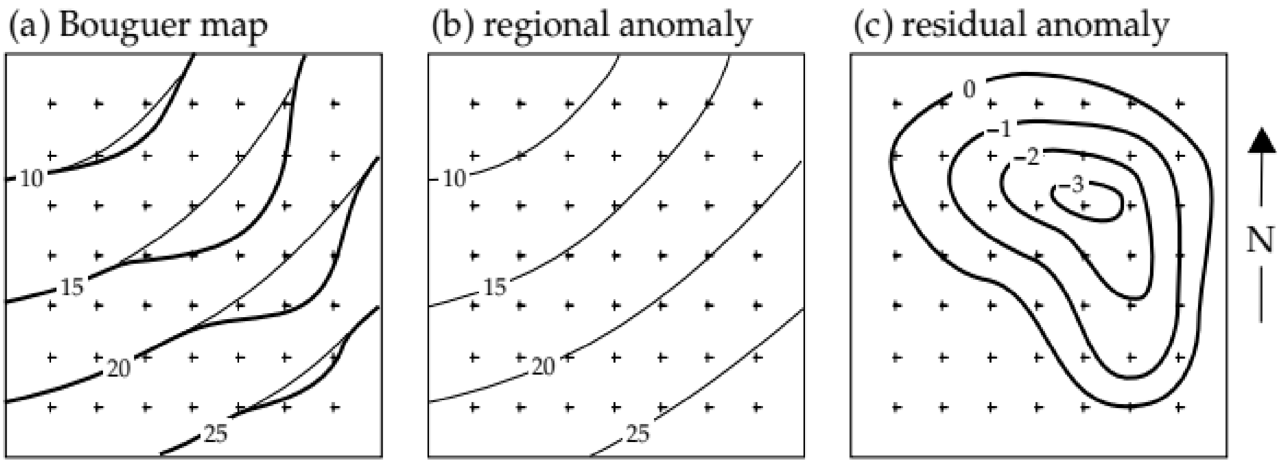


Figure IV.20: The total signal is shown at the top. The regional anomaly is in the middle and the residual signal at the bottom (gtotal = gregional + gresidual)

The figure above shows a Bouguer anomaly profile. It is clear that short-wavelength anomalies are superimposed on a large trend, which can be approximated here by a low-order polynomial. As we have just seen, the short-wavelength anomaly corresponds to a field that we can try to define, while the trend corresponds to a distribution of fields that is virtually impossible to interpret since we do not know the entire signal. This latter signal is called the regional anomaly.

Total anomaly = Regional anomaly + Residual anomaly



II.8.3.3 Some Simple Processing: Extensions and Derivatives

When interpreting gravimetric anomaly maps, it can be useful to perform some preliminary or complementary processing. The purpose of this processing is to better separate anomalies, to better define the depths of sources, and to represent geological boundaries such as contacts or faults, etc. We will briefly examine two of these: **extensions** and **derivatives**.

a) Extension

Extending an anomaly observed on a given surface involves calculating the shape and of this anomaly on a surface located at a different altitude. If the calculation is performed on a surface higher than the observation surface, it is an upward extension, and otherwise a downward extension.

This operation allows for the comparison of data acquired at different altitudes, for example, on the topographic surface and from an airplane. It can also be shown that a downward extension is equivalent to filtering long wavelengths (high-pass filter), while an upward extension is equivalent to filtering short wavelengths (low-pass filter). Note also that the downward extension is difficult to obtain because numerical instabilities can occur during the calculation; in particular, the surface on which the downward extension is performed must remain above the sources.

Figures II.22 and II.23 show examples of extensions.

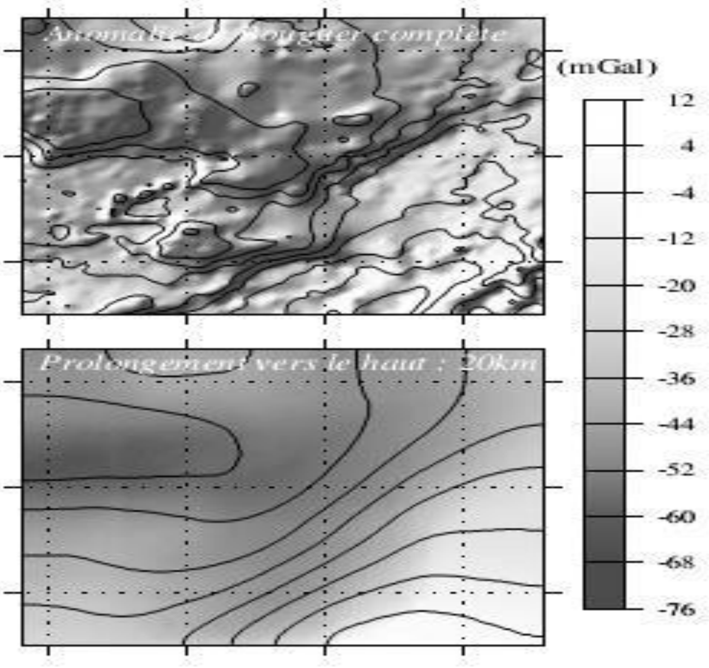


Figure IV.22: Top, observed Bouguer anomaly and bottom, the same anomaly extended upwards to an altitude of 2.0 km

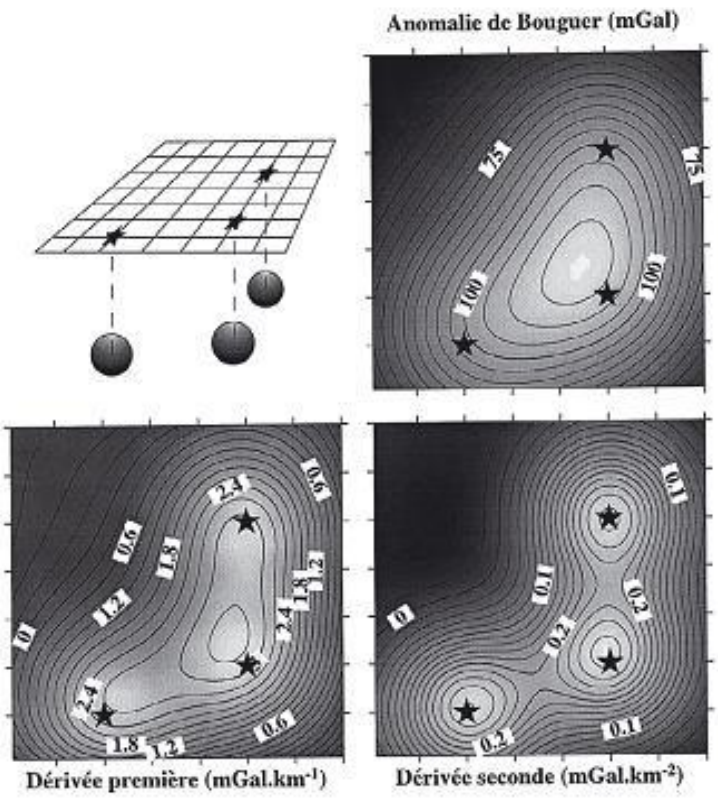


Figure II.23: Bouguer anomaly created by closely spaced spherical sources (top) and vertical derivative maps. The higher the order of the derivative, the better the separation of the anomalies