Each theorem comes with a **short theoretical explanation** and an **example**.

GEOMETRY THEOREMS

1. Sum of Angles in a Triangle**

Theorem:

The three interior angles of any triangle add up to **180°**.

Proof (idea):

Draw a line parallel to the base through the top vertex.

The alternate interior angles form a straight line (180°).

Example:

If ∠A = 70°, ∠B = 50°, then

∠C = 180° - (70° + 50°) = 60°.

⍰ **Answer: 60°.**

2. Exterior Angle Theorem**

Theorem:

The exterior angle of a triangle equals the sum of the two opposite interior angles.

Proof (idea):

{Exterior} {Interior at that vertex} = 180°,

and the sum of all three interior angles = 180°.

Subtract → exterior = sum of other two.

Example:

If opposite angles = 50° and 60°,

then exterior angle = 50° + 60° = **110°**.

⍰ **Answer: 110°.**

3. Isosceles Triangle Theorem**

Theorem:

In an isosceles triangle, angles opposite the equal sides are equal.

Proof (idea):

If (AB = AC), draw a line from A to the midpoint of BC.

It divides the triangle into two equal right triangles.

Example:

If ∠B = 45°, then ∠C = 45° too.

Base angles are equal

4. Circle Theorem (Angle at the Centre)**

Theorem:

The angle at the **centre** is **twice** the angle at the **circumference** standing on the same arc.

Example:

If the angle at the centre = 100°,

then at the circumference = (\frac{100°}{2} = 50°).

⍰ **Answer: 50°.**

SET THEORY THEOREMS

5. Subset Definition**

Theorem:

If every element of set A is also in set B, then A is a **subset** of B.

A ⊆ B

Example:

A = {1, 2}, B = {1, 2, 3, 4}

Every element of A is in B → A ⊆ B

⍰ **True.**

6. Union of Sets**

Theorem:

The **union** of A and B, written (A ∪ B), is the set of elements that are in A **or** in B (or in both).

Example:

A = {1, 2, 3}, B = {3, 4, 5}

Then (A ∪ B = {1, 2, 3, 4, 5}).

⍰ **All elements combined.**

7. Intersection of Sets**

Theorem:

The **intersection** of A and B, written (A ∩ B), is the set of elements common to both.

Example:

A = {2, 3, 5}, B = {1, 3, 5, 7}

Then (A ∩ B = {3, 5}).

⍰ **Common elements.**

8. Complement of a Set**

Definition:

The **complement** of a set A, written (A'), is the set of all elements **not in A** (with respect to a universal set U).

Example:

U = {1, 2, 3, 4, 5}, A = {1, 2, 3}

Then (A' = {4, 5}).

□ **All not in A.**

9. De Morgan's Laws**

Theorem:

1□ ((A ∪ B)' = A' ∩ B')

2 ((A ∩ B)' = A' ∪ B')

Proof (idea):

Use logic: "not (A or B)" = "not A and not B."

Example:

U = {1, 2, 3, 4, 5}, A = {1, 2}, B = {2, 3}

Then:

(A ∪ B = {1, 2, 3} → (A ∪ B)' = {4, 5})

and

(A' = {3, 4, 5}, B' = {1, 4, 5} → A' ∩ B' = {4, 5})

▪ **Equal → theorem true.**

NUMBER THEORY THEOREMS

10. Even and Odd Numbers

Theorem:

* Even + Even = Even

* Odd + Odd = Even

* Even + Odd = Odd

Proof (idea):

Even = 2n, Odd = 2n + 1 → use algebraic addition.

Example:

4 + 6 = 10 (even)

3 + 5 = 8 (even)

6 + 7 = 13 (odd)

▪ **True.**

11. Divisibility by 3

Theorem:

A number is divisible by 3 if the **sum of its digits** divisible by 3.

Example:

153 → 1 + 5 + 3 = 9 → divisible by 3 → 153 is divisible by 3.

▪ **True.**

12. Prime Numbers**

Definition:

A prime number has only two factors: **1 and itself.**

Examples:

2, 3, 5, 7, 11 are prime.

4 is not (1×4 and 2×2).

▪ **Defined.**

13. Square of Even and Odd Numbers**

Theorem:

* The square of an even number is even.

* The square of an odd number is odd.

Example:

Even: (4^2 = 16) (even)

Odd: (5^2 = 25) (odd)

▪ **True.**

Geometry, Set Theory & Number Theory – Simple Theorems and Problems with Solutions

Geometry

Vertically Opposite Angles

When two lines intersect, opposite angles are equal.

Example: If one angle = 120°, opposite angle = 120°.

Complementary Angles

Two angles are complementary if their sum is 90°.

Example: 35° + 55° = 90°.

Supplementary Angles

Two angles are supplementary if their sum is 180°.

Example: 100° + 80° = 180°.

Perpendicular Lines

Two lines are perpendicular if they meet at 90°.

Example: Line AB ■ line CD → angle = 90°.

Parallel Lines

Two lines are parallel if they never meet.

Example: Lines on notebook paper are parallel.

Rhombus Property

All sides equal; diagonals bisect at 90°.

Example: Diagonals 10 cm and 8 cm → halves 5 cm, 4 cm.

Rectangle Property

Opposite sides equal; all angles 90°.

Example: Perimeter = 2(8 + 4) = 24 cm.

Square Property

All sides equal; all angles 90°.

Example: Side = 6 cm → perimeter = 24 cm.

Parallelogram Property

Opposite sides and angles equal.

Example: One side = 10 cm → opposite side = 10 cm.

Area of a Triangle

Area = ½ × base × height.

Example: Base = 8, height = 5 → area = 20 cm².

Set Theory

Universal Set

Contains all elements under consideration.

Example: A={1,2}, B={2,3} → U={1,2,3}.

Cardinal Number

n(A) = number of elements in set A.

Example: A={2,4,6,8} → n(A)=4.

Equal Sets

Sets are equal if they have the same elements.

Example: A={1,2,3}, B={3,2,1} → Equal.

Power Set

Set of all subsets of A.

Example: A={1,2} → P(A)={∅,{1},{2},{1,2}}.

Intersection

Common elements of A and B.

Example: A={x<5},B={x>3} → {4}.

Union

+ Set Theorv + Number Theorv**

⍰ **Base angles are equal.**

All elements from A, B, and C.

Example: $A=\{1,2,3\}, B=\{3,4,5\}, C=\{5,6\} \rightarrow \{1,2,3,4,5,6\}$.

Complement

Elements in U but not in A.

Example: $U=\{1,2,3,4,5\}, A=\{1,2,3\} \rightarrow A'=\{4,5\}$.

Symmetric Difference

Elements in A or B, not both.

Example: $A=\{1,2,3\}, B=\{3,4,5\} \rightarrow \{1,2,4,5\}$.

Number Theory

Divisibility by 5

Ends with 0 or 5.

Example: 35 → divisible.

Divisibility by 10

Ends with 0.

Example: 120 → divisible.

Divisibility by 2

Even number.

Example: 246 → divisible.

Prime Numbers (1–20)

Numbers divisible only by 1 and itself.

Example: 2,3,5,7,11,13,17,19.

HCF

Highest Common Factor of 18,24=6.

Example: Common factors $=2\times3=6$.

LCM

Lowest Common Multiple of 6,8=24.

Example: $LCM=2^3\times3=24$.

Perfect Cube

Number $= n^3$.

Example: $8=2^3, 27=3^3, 64=4^3$.

Square Root

Number that squared gives x.

Example: $\sqrt{49}=7$.

Odd Number Rule

Sum of 2 odd numbers = even.

Example: $3+5=8$.

Even Number Rule

Product of 2 even numbers = even.

Example: $4\times6=24$.