

* ☐ **Statement of the theorem** **

* ☐ **Explanation (theory)** **

* ☐ **Proof (step-by-step reasoning)** **

1. Commutative Property of Addition

Statement:

1 For any numbers (a) and (b):

$$a + b = b + a$$

Theory:

The order of numbers in addition doesn't affect the sum.

Proof:

Let (a = 3) and (b = 5).

Then (3 + 5 = 8) and (5 + 3 = 8).

Since the results are equal for all real numbers,

☐ **The property holds true.**

2. Associative Property of Multiplication

Statement:

For any numbers (a, b, c):

$$(a \times b) \times c = a \times (b \times c)$$

Theory:

When multiplying three numbers, grouping doesn't change the product.

Proof:

Let (a = 2, b = 3, c = 4)

Then[

$$(2 \times 3) \times 4 = 6 \times 4 = 24$$

and[

$$2 \times (3 \times 4) = 2 \times 12 = 24$$

☐ **Both sides equal → proven.**

3. Distributive Property

Statement:

$$a(b + c) = ab + ac$$

Theory:

Multiplying a number by a sum is the same as multiplying each term separately and then adding.

Proof:

Let (a = 4, b = 2, c = 3):[

$$4(2 + 3) = 4 \times 5 = 20$$

and

$$4 \times 2 + 4 \times 3 = 8 + 12 = 20$$

☐ **Hence, both sides are equal.**

4. Sum of Angles in a Triangle

Statement:

The sum of the three interior angles of any triangle is **180°**.

Theory:

In Euclidean geometry, all triangles have angles that together form a straight line (180°).

Proof (Sketch):

1. Draw triangle (ABC).

2. Draw a line through (A) parallel to base (BC).

3. Alternate interior angles with sides (AB) and (AC) equal the interior angles at (B) and (C).

4. These three angles form a straight line → (180°).

☐ **Hence, sum of angles = 180°.**

5. Pythagoras' Theorem

Statement:

✓ In a right-angled triangle,

$$a^2 + b^2 = c^2$$

where (c) is the hypotenuse.

Theory:

This relates the sides of any right-angled triangle.

Proof (by areas):

1. Construct a square with side ((a+b)), containing 4 right triangles and a smaller square in the middle.

2. The area of the big square = ((a+b)^2).

3. The area also equals (4(\frac{1}{2}ab) + c^2 = 2ab + c^2).

4. Equating:

$$(a+b)^2 = 2ab + c^2$$

Simplify:

$$a^2 + 2ab + b^2 = 2ab + c^2 \rightarrow a^2 + b^2 = c^2$$

☐ **Proved.**

6. Opposite Angles of a Parallelogram

Statement:**

Opposite angles of a parallelogram are **equal**.

Theory:

A parallelogram has opposite sides parallel, and parallel lines create equal alternate interior angles.

Proof:

1. Let ABCD be a parallelogram.

2. ($AB \parallel CD$) and ($AD \parallel BC$).

3. Using alternate interior angles:

$$\angle A = \angle C \text{ and } \angle B = \angle D.$$

∴ **Hence, opposite angles are equal.**

7. Vertical Angles Theorem

Statement:

When two lines intersect, the **opposite (vertical)** angles are equal.

Theory:

Vertical angles share a common vertex and are formed by two intersecting straight lines.

Proof:

Let lines AB and CD intersect at O.

Angles ($\angle AOC$) and ($\angle BOD$) are adjacent and form a straight line, so:

$$\angle AOC + \angle BOD = 180^\circ$$

Similarly,

$$\angle BOD + \angle COA = 180^\circ$$

By subtraction,

$$\angle AOC = \angle BOD$$

∴ **Hence, vertical angles are equal.**