Dynamics of Perfect Incompressible Fluids

1. Flow Description Methods

There are two main ways to study fluid flow. Take the example of smoke flowing out of a chimney (Figure 1). The first method consists of following the same particle "A" of smoke as it moves, and measuring its temperature at each moment—this is called the Lagrangian method. The second method fixes a point "O" in space and measures the temperature of the particles that pass through it—this is the Eulerian method. For a single fixed position (x_0, y_0, z_0), we obtain a sequence of temperatures T_{01}, T_{02}, T_{03}... measured at times t_1, t_2, t_3..., etc. This second method is the most common and effective in most cases.
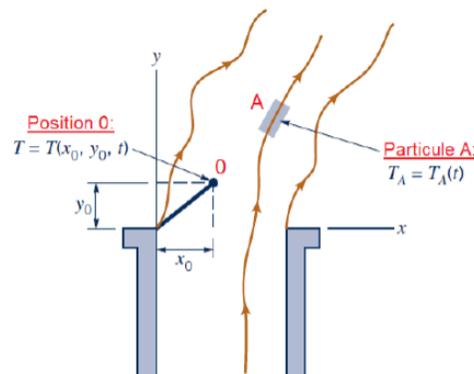

Fig 1. Lagrangian and Eulerian description of the flow of smoke coming out of a chimney

2. Steady and Unsteady Flow

If, at every point in space, the flow (velocity, pressure, density, etc.) does not depend on time, the flow is called steady (or stationary). If it varies with time, it is called unsteady (or non-stationary).

3. Conservation of Mass

The principle of mass conservation is one of the most fundamental principles in nature. For closed systems, it means that mass remains constant during a process. For open systems, however, mass can cross the system's boundaries, so we must account for the amount entering and leaving.

Reference: Course notes MDF "Dr. Z. Alloui"; Fluid Mechanics Fundamentals and Applications "Cengel & Cimbala"

4. . Volumetric Flow Rate

The volumetric flow rate at a point is the volume of fluid passing that point per second. If during a time interval Δt a volume ΔV passes, then the volumetric flow rate Q (in m³/s) is given by:

$Qv = \Delta V / \Delta t$

5. Mass Flow Rate

The mass flow rate at a point is the mass of fluid passing that point per second. If during Δt a mass Δm passes, then the mass flow rate Qm (in kg/s) is:

$Qm = \Delta m / \Delta t$

For a fluid flowing through a pipe (Figure 2), it can also be expressed as:

$Qm = \rho\, S\, \dot{v}$ and $Qv = S\, v$

where:

S = cross-sectional area (m²)

ρ = fluid density (kg/m³)

v = mean velocity (m/s)

Mass conservation implies that the mass flow rate is equal at sections A and B:

$$\dot{m}_A = \dot{m}_B \Leftrightarrow \rho_A\, S_A\, v_A = \rho_B\, S_B\, v_B$$
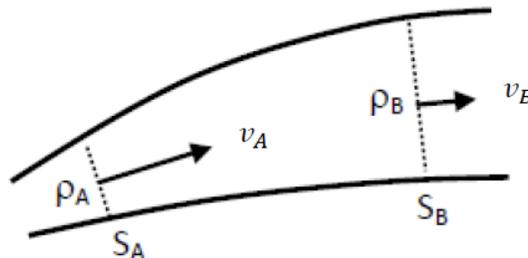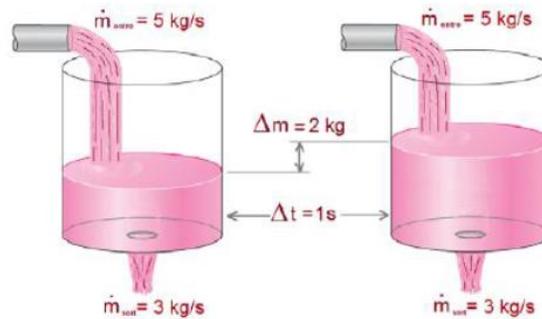

Fig 2. flow in a pipe

6. Mass Conservation Principle

The net mass transfer entering or leaving a system during a time Δt equals the net

change of total mass within the system. For example, if water enters a tank at 5 kg/s and leaves at 3 kg/s, there is an accumulation of 2 kg/s inside.

$$\dot{m}_{in} - \dot{m}_{out} = dm/dt$$

Fig.3 Mass preservation principle for an open container

The three-dimensional form of the mass conservation (continuity) equation is:

$$\partial\rho/\partial t + \partial(\rho u)/\partial x + \partial(\rho v)/\partial y + \partial(\rho w)/\partial z = 0$$

For incompressible flow (ρ constant):

$$\partial u/\partial x + \partial v/\partial y + \partial w/\partial z = 0$$

7. Bernoulli's Theorem

According to the conservation of energy, the sum of kinetic, pressure, and potential energies (i.e., total mechanical energy) remains constant:

$$v^2/2 + P/\rho + gz = constant$$

Dividing by g gives:

$$v^2/(2g) + P/(\rho g) + z = constant$$

Multiplying by ρ gives:

$$\rho v^2/2 + P + \rho gz = constant$$

Simplifications:

• Applies along a streamline

• Incompressible fluid (ρ = const)

• Steady flow ($\partial v/\partial t = 0$)

• Negligible viscous effects

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8. Flow Between Two Points Without Work Exchange

For a perfect, incompressible fluid without machines (pumps/turbines) between two points (1) and (2) along a streamline:

$$\rho/2 \ (v_2{}^2 - v_1{}^2) + (P_2 - P_1) + \rho g(z_2 - z_1) = 0$$

Applications

Example 1:

A 2.54 cm diameter pipe is used to fill a 75 L water bucket. If the mean water velocity in the pipe is 2.44 m/s, determine:

a) The volumetric (L/s) and mass (kg/s) flow rates

b) The time required to fill the bucket

Answer: a) $Q = 1.236$ L/s, $\dot{m} = 1.236$ kg/s, b) $\Delta t = 61$ s

Example 2:

A swimming pool with diameter $D = 10$ m contains 2 m of water. It is emptied using a 25 m long horizontal pipe of diameter $d = 3$ cm. Determine the maximum discharge flow rate Q (L/s).

Answer: Qmax = 4.43 L/s