# Fluid Mechanics - Chapter 2: Fluid Statics

## Definition of Pressure

Pressure is defined as a normal force exerted by a fluid per unit area. We speak of pressure when dealing with a gas or a liquid, while in solids we speak of normal stress. Since pressure is defined as a force per unit area, its SI unit is the newton per square meter ($N/m^2$), also called the pascal (Pa). That is: 1 Pa = 1 $N/m^2$.

$$P = F / S$$

Because the pascal is small for most practical pressures, multiples such as the kilopascal (1 kPa = $10^3$ Pa) and the megapascal (1 MPa = $10^6$ Pa) are commonly used. Other pressure units include the bar, the standard atmosphere (atm), and the kilogram-force per square centimeter ($kgf/cm^2$).

1 bar = $10^5$ Pa = 0.1 MPa = 100 kPa
1 atm = 101,325 Pa = 101.325 kPa = 1.01325 bar
1 $kgf/cm^2$ = $9.807 \times 10^4$ Pa = 0.9807 bar = 0.9679 atm
1 bar = 14.504 psi (pound-force per square inch)

## 1. Absolute and Gauge Pressure

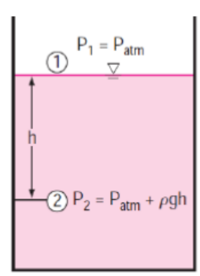
The actual pressure at a given point is called absolute pressure and is measured relative to a perfect vacuum. Most pressure-measuring devices are calibrated to read zero when exposed to the atmosphere. Thus, they indicate the difference between the absolute pressure and the local atmospheric pressure, called gauge pressure:

$P_{gauge} = P_{absolute} - P_{atmospheric}$

## 2. Fundamental Law of Fluid Statics

In a stationary fluid, pressure does not change horizontally, but in a gravitational field it increases with depth:

$P_2 = P_{atm} + \rho gh$
$P_{gauge} = \rho gh$

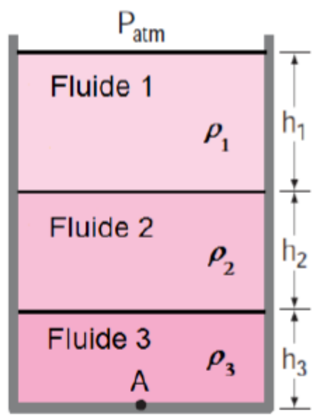



Differential form: $dP/dz = -\rho g$

## 3. Pressure in Superimposed Immiscible Fluids

Many engineering problems involve several immiscible fluids stacked one on another with different densities. For a tank containing three non-miscible liquids with densities $\rho_1 < \rho_2 < \rho_3$, the pressure at the bottom is:

$$P_A = P_{atm} + \rho_1 g h_1 + \rho_2 g h_2 + \rho_3 g h_3$$




## 4. Equipotential (Level) Surfaces

A level surface is the set of points in a fluid that are under the same pressure. Integrating $dP/dz = -\rho g$ gives $P = -\rho g z + \text{constant}$. If P is constant, then z is constant; therefore, level surfaces are horizontal planes.

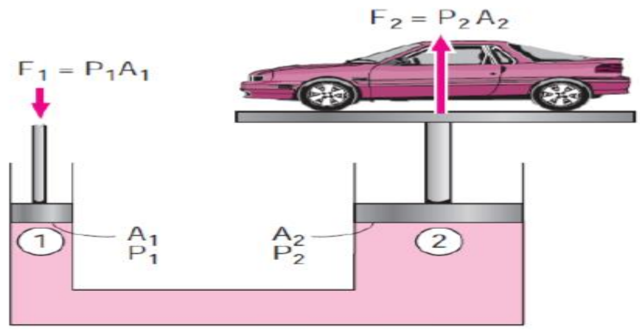
## 5. Pascal's Theorem

Pressure in a fluid remains constant horizontally. When pressure is increased at one point, it increases equally throughout the fluid.

Pascal's Law: In an incompressible fluid at rest, any pressure increase applied at one point is transmitted undiminished throughout the fluid.

This principle underlies hydraulic systems such as jacks, lifts, presses, and aircraft hydraulic controls. For pistons of areas $S_1$ and $S_2$ subjected to forces $F_1$ and $F_2$ respectively:

$$F_1/S_1 = F_2/S_2 \rightarrow F_2 = F_1 \times (S_2/S_1)$$



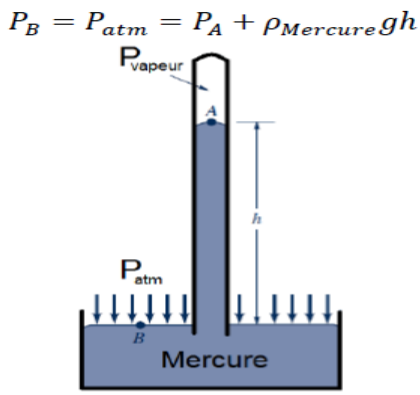
## 6. Pressure Measurement Instruments

Because pressure is an important fluid property, many instruments exist to measure it. Absolute pressure is measured relative to a perfect vacuum, while gauge pressure is measured relative to local atmospheric pressure.

### 6.1 Mercury Barometer

Atmospheric pressure is commonly measured with a mercury barometer, invented by Torricelli in 1643:

$P_B = P_{atm} = P_A + \rho_{mercury} \times g \times h$




### 6.2 Manometers

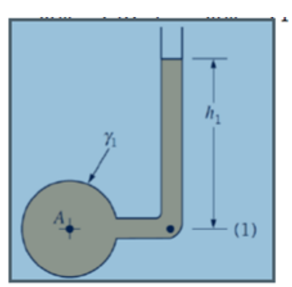
A manometer measures pressure using a column of liquid in a vertical or inclined tube. Common types include the piezometer, U-tube manometer, and inclined-tube manometer.

#### *Piezometer*

A transparent tube, open to the atmosphere at one end and connected to the fluid system at the other:

$P_A = P_{atm} + \rho_1 g h_1 = P_{atm} + \gamma_1 h_1$.

Limitations: only for pressures above atmospheric, suitable for low pressures, and only for liquids (not gases).



### *U-Tube Manometer*

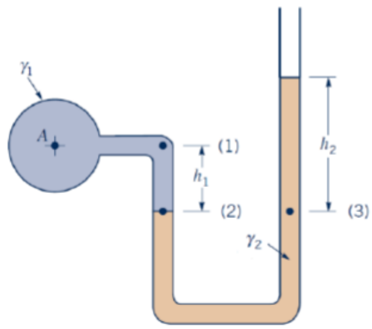
A U-shaped transparent tube containing one or more fluids:

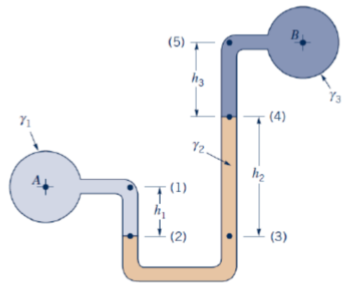
$P_A = P_{atm} + \gamma_2 h_2 - \gamma_1 h_1$.

If fluid (1) is a gas, $\gamma_1 h_1$ can be neglected, giving $P_A = P_{atm} + \gamma_2 h_2$.

A U-tube manometer can also measure pressure differences between two points:

$P_A - P_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$.




## Exercises

### Exercise 1

A closed tank contains 1.5 m of oil ($\gamma$_oil = 8720 N/$m^3$), 1 m of water ($\gamma$_water = 9790 N/$m^3$), and 0.2 m of mercury ($\gamma$_Hg = 133,100 N/$m^3$). If the pressure at the bottom is P_bottom = 60 kPa, find the air pressure at the top.

Answer: $P_{air}$ = 10,500 Pa

### Exercise 2

Air is pressurized in a piston-cylinder device by a spring and the piston's weight. The piston area is $35 \times 10^{-4}$ $m^2$, its mass is 4 kg, and atmospheric pressure is 95 kPa. If the spring exerts a 60 N force on the piston, determine the air pressure inside the cylinder.

Answer: P = 123.4 kPa