

Material 3 (Initialisation of the simplex algorithm, the two-phase method, the big-M method)

Exercise 1. Solve the following linear programme using the two-phase simplex method:

$$\text{Min } Z = 3x_1 + 2x_2$$

$$\begin{cases} 3x_1 + x_2 \geq 3 \\ 4x_1 + 3x_2 \geq 6 \\ x_1 + x_2 \leq 3 \\ x_1, x_2 \geq 0 \end{cases}$$

Exercise 2. Solve the linear programme (P) using the two-phase simplex algorithm.

$$\text{Max } Z = 2x_1 + 3x_2 + x_3$$

$$\begin{cases} x_1 + x_2 + x_3 \leq 40 \\ 2x_1 + x_2 - x_3 \geq 10 \\ -x_1 + x_3 \geq 10 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

Exercise 3 Solve the linear programme (P) using the two-phase simplex algorithm.

$$\text{Max } Z = 3x_1 + 4x_2 + 5x_3$$

$$\begin{cases} x_1 + 2x_2 + 3x_3 \geq 5 \\ 2x_1 + 2x_2 + x_3 \geq 6 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

Exercise 4. Show that the following linear problem does not have feasible solution (suggestion: use the Big-M method).

$$\text{Max } Z = 12x_1 + 15x_2 + 10x_3$$

$$\begin{cases} x_1 + x_2 + 2x_3 \geq 10 \\ 15x_1 + 6x_2 - 5x_3 \leq 30 \\ x_1 + 3x_2 + 5x_3 \leq 18 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

Exercise 5. Solve the linear programme (P) using the Big M method.

$$\text{Min } W = 2x_1 + 1x_2 + 4x_3$$

$$\begin{cases} x_1 + x_2 + 2x_3 = 3 \\ 2x_1 + x_2 + 3x_3 = 5 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$