

Tutorial N°2 (Simplex algorithm)

Exercise 1. What is the largest and smallest value that the function can take:

$$Z = x_1 - x_2 + x_3.$$

Given that the variables must be positive or zero, their sum must be less than or equal to 8, with x_2 less than or equal to 1.

Formulate the two corresponding linear programs and solve them using the simplex algorithm.

Exercise 2. Consider the following linear program (LP):

$$\text{Max } Z = x_1 + 2x_2$$

$$\begin{cases} -3x_1 + 2x_2 \leq 2 \\ -x_1 + 2x_2 \leq 4 \\ x_1 + x_2 \leq 5 \\ x_1, x_2 \geq 0 \end{cases}$$

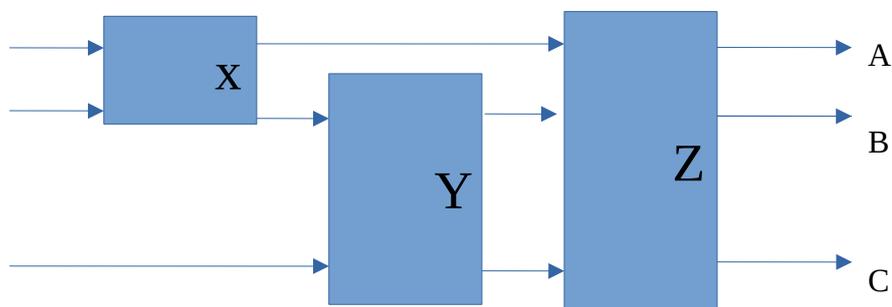
1. Solve the linear program (LP) using the graphical method.
2. Solve the linear program (LP) using the simplex method.

Exercise 3. A factory manufactures three products, A, B, and C. Each product is processed sequentially by machines X, Y, and Z as follows:

Product A: processed by machines X and Z

Product B: processed by machines X, Y, and Z

Product C: processed by machines Y and Z



Graphical representation of the manufacturing process

Each unit of the product requires one unit of machine capacity. The daily capacities of the machines are: $X = 100$, $Y = 200$, and $Z = 400$.

The profits for each unit of A, B, and C are 3, 4, and 2, respectively.

1. Provide the mathematical formulation of the manufacturing process that will yield the maximum profit for this factory.
2. Solve the linear program obtained to determine the optimal manufacturing plan.

Exercise 4. Solve the following linear problem using the simplex method.

$$\text{Max } Z = 3x_1 + 2x_2 + 4x_3$$

$$\begin{cases} x_1 + x_2 + 2x_3 \leq 4 \\ 2x_1 + 3x_3 \leq 5 \\ 2x_1 + x_2 + 3x_3 \leq 7 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

Exercise 5. Consider the following linear program:

$$\text{Max } Z = -5x_1 + 5x_2 + 13x_3$$

$$\begin{cases} -x_1 + x_2 + 3x_3 \leq 20 \\ 12x_1 + 4x_2 + 10x_3 \leq 90 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

- 1- Solve the problem using the simplex algorithm.
- 2- If we increase the second member of the first constraint from 20 to 30, what will be the optimal solution below this value?
- 3- What is the validity interval of the optimal basis of the problem, changing the second member of the first constraint?