Basic Geometry Postulates

1. **Postulate of Points**: Through any two distinct points, there is exactly one straight line.

   * **Example**: Given points A and B, there is one and only one line that passes through both points.

2. **Postulate of Lines**: A line segment can be extended indefinitely in both directions to form a line.

   * **Example**: The segment AB can be extended to form the line (\overleftrightarrow{AB}).

3. **Postulate of Intersection of Two Lines**: Two distinct lines intersect at most at one point.

   * **Example**: If lines L and M intersect, they will do so at exactly one point, say P.

4. **Postulate of Flatness**: Through any three non-collinear points, there is exactly one plane.

   * **Example**: Points A, B, and C that are not all on the same line determine a unique plane.

5. **Postulate of Plane Intersection**: If two planes intersect, their intersection is a line.

   * **Example**: If planes P and Q intersect, they do so along a line L.

6. **Postulate of Points in a Plane**: A plane contains at least three non-collinear points.

   * **Example**: Three points that do not lie on the same line determine a plane.

7. **Postulate of Parallel Lines**: If two parallel lines are cut by a transversal, the corresponding angles are equal.

   * **Example**: If lines L and M are parallel, and they are cut by transversal T, then the corresponding angles formed are equal.

8. **Postulate of Parallel Planes**: If two parallel planes are cut by a transversal, the intersections with the transversal are parallel.

   * **Example**: If planes P and Q are parallel, and they are cut by transversal R, the lines of intersection are parallel.

9. **Postulate of Distance**: The distance between two points is a positive real number.

   * **Example**: The distance between points A(1, 2) and B(4, 6) is (\sqrt{(4-1)^2 + (6-2)^2} = 5).

10. **Postulate of Right Angles**: All right angles are equal.

    * **Example**: Any two right angles are congruent.

11. **Postulate of Midpoint**: The midpoint of a segment is the point that divides the segment into two equal parts.

    * **Example**: The midpoint of segment AB is the point M such that AM = MB.

Algebraic Postulates

12. **Commutative Property of Addition**: (a + b = b + a).

    * **Example**: (3 + 5 = 5 + 3 = 8).

13. **Commutative Property of Multiplication**: (a \times b = b \times a).

    * **Example**: (4 \times 6 = 6 \times 4 = 24).

14. **Associative Property of Addition**: ((a + b) + c = a + (b + c)).

    * **Example**: ((2 + 3) + 4 = 2 + (3 + 4) = 9).

15. **Associative Property of Multiplication**: ((a \times b) \times c = a \times (b \times c)).

    * **Example**: ((2 \times 3) \times 4 = 2 \times (3 \times 4) = 24).

16. **Distributive Property**: (a(b + c) = ab + ac).

    * **Example**: (2(3 + 4) = 2 \times 3 + 2 \times 4 = 14).

17. **Identity Property of Addition**: (a + 0 = a).

    * **Example**: (5 + 0 = 5).

I can definitely add some problems that involve both **algebraic concepts** (like variables and equations) and **simple geometric concepts** (like perimeter, area, and angles).
Here are 10 new problems, mixing algebra and geometry, in the same brief format.

Algebra & Geometry Word Problems (Set 5)

No.	Problem	Setup	Answer
1	A **triangle's** perimeter is 33 cm. The sides are x, x+2, and x+4. Find the length of the shortest side.	x + (x+2) + (x+4) = 33	3x + 6 = 33; 3x = 27; x = \mathbf{9} cm.
2	The **length of a rectangle** is 5 more than its width. If the perimeter is 46 units, find the width (w).	2w + 2(w + 5) = 46	4w + 10 = 46; 4w = 36; w = \mathbf{9} units.
3	The **area of a square** is 64 sq. units. If the side length is 2x, find the value of x.	(2x)^2 = 64	4x^2 = 64; x^2 = 16; x = \mathbf{4}
4	A line segment of length 20 is divided into two parts. One part is 4 times the other. Find the length of the shorter part.	x + 4x = 20	5x = 20; x = \mathbf{4}
5	Two **complementary angles** (sum to 90^\circ) have measures x and 2x. Find the measure of the smaller angle.	x + 2x = 90	3x = 90; x = \mathbf{30^\circ}
6	The **base of a parallelogram** is x, and its height is 4. If the area is 28, find the base.	4x = 28	x = \mathbf{7}
7	An isosceles **triangle** has two equal sides of length 2x. The third side is 8. If the perimeter is 24, find x.	2x + 2x + 8 = 24	4x = 16; x = \mathbf{4}
8	The measure of an angle is 10 less than its **supplement** (sum to 180^\circ). Find the	x = (180 - x) - 10	2x = 170; x = \mathbf{85^\circ}
9	A **trapezoid** has a height of 5 and bases of length x and x+6. If the area is 50, find the length of the shorter base.	\frac{1}{2}(b_1 + b_2)h = 50 \implies \frac{1}{2}(x + x + 6)5 = 50	5(2x + 6) = 100; 10x + 30 = 100; 10x = 70; x = \mathbf{7}
10	The **diameter of a circle** is 3x. If the diameter is 18 cm, find x.	3x = 18	x = \mathbf{6}

Let me know if you'd like more problems or another set!

Geometry, Set Theory & Number Theory – Simple Theorems and Problems with Solutions

Geometry

Vertically Opposite Angles

When two lines intersect, opposite angles are equal.

Example: If one angle = 120°, opposite angle = 120°.

Complementary Angles

Two angles are complementary if their sum is 90°.

Example: 35° + 55° = 90°.

Supplementary Angles

Two angles are supplementary if their sum is 180°.

Example: 100° + 80° = 180°.

Perpendicular Lines

Two lines are perpendicular if they meet at 90°.

Example: Line AB ■ line CD → angle = 90°.

Parallel Lines

Two lines are parallel if they never meet.

Example: Lines on notebook paper are parallel.

Rhombus Property

All sides equal; diagonals bisect at 90°.

Example: Diagonals 10 cm and 8 cm → halves 5 cm, 4 cm.

Rectangle Property

Opposite sides equal; all angles 90°.

Example: Perimeter = 2(8 + 4) = 24 cm.

Square Property

All sides equal; all angles 90°.

Example: Side = 6 cm → perimeter = 24 cm.

Parallelogram Property

Opposite sides and angles equal.

Example: One side = 10 cm → opposite side = 10 cm.

Area of a Triangle

Area = ½ × base × height.

Example: Base = 8, height = 5 → area = 20 cm².

Set Theory

Universal Set

Contains all elements under consideration.

Example: A={1,2}, B={2,3} → U={1,2,3}.

Cardinal Number

n(A) = number of elements in set A.

Example: A={2,4,6,8} → n(A)=4.

Equal Sets

Sets are equal if they have the same elements.

Example: A={1,2,3}, B={3,2,1} → Equal.

Power Set

Set of all subsets of A.

Example: A={1,2} → P(A)={∅,{1},{2},{1,2}}.

Intersection

Common elements of A and B.

Example: A={x<5},B={x>3} → {4}.

Union

1. Parentheses

The first step to the order of operations is parentheses. This means that when we're solving an expression with multiple operations, we first solve the operations within parentheses (). Here's a simple example to illustrate.

$3 \times (4 + 2)$

- First, we solve the expression within the parentheses: $4 + 2 = 6$, so we get 3×6.
- Then, we multiply 3 by the result of the expression inside the parentheses (which, as we've seen, is 6): 3×6.
- Finally, we get the answer 18.

Sometimes, math expressions contain a set of parentheses inside another set. To solve them, **start by solving the operations inside the innermost parentheses**, then work your way outward.

For example: $\{4 \times [5 + (6 - 3)]\} - (8 - 2)$

- First, we subtract numbers inside the innermost parentheses: $6 - 2 = 3$, so we get $\{4 \times [5 + 3]\} - (8 - 2)$.
- Then, add 5 to the answer inside the square brackets: $5 + 3 = 8$, so we get $\{4 \times 8\} - (8 - 2)$.
- Next, multiply the result by 4 inside the curly brackets: $4 \times 8 = 32$, so we get $32 - (8 - 2)$.
- Now, solve the expression inside the parantheses: $8 - 2 = 6$, so we get $32 - 6$.
- Finally, subtract the result of step 4 from the result of step 3: $32 - 6 = 26$

2. Exponents

The second step in the order of operations is exponents. Exponents, also known as indices, show how many times a number is multiplied by itself.

To illustrate, 3^4 is an exponent form that means multiply 3 by itself 4 times, or $3 \times 3 \times 3 \times 3$.

Let's look at this example:

$2^2 + 4 \times 2$

- First, we look at the exponent 2^3 which means we multiply 2 by itself 3 times, or $2 \times 2 \times 2 = 8$, so we get: $8 + 4 \times 2$.
- Then we multiply: $4 \times 2 = 8$, so we get $8 + 8$.
- Finally, we add: $8 + 8 = 16$.

Refresh Your Memory: What Does Squared Mean in Math?

3. Multiplication & Division

In the order of operations, multiplication and division are prioritized equally and are performed **from left to right** as they appear in the expression. This is why we consider them together as the third step in PEMDAS.

Let's see that in practice with: $8 \div 2 \times 4 + 3$

- First, we perform the division because it appears first, before the multiplication: $8 \div 2 = 4$, so we get $4 \times 4 + 3$.
- Then, we perform the multiplication: $4 \times 4 = 16$, so we get $16 + 3$.
- Next, we perform the addition: $16 + 3 = 19$.
- Finally, we get the result 19.

PEMDAS Have the same priority level.
PEDMAS
parentheses / Exponents / Division / Multiplication / Addition / subtraction

4. Addition & Subtraction

As the last steps of PEDMAS, we work on the addition and subtraction.

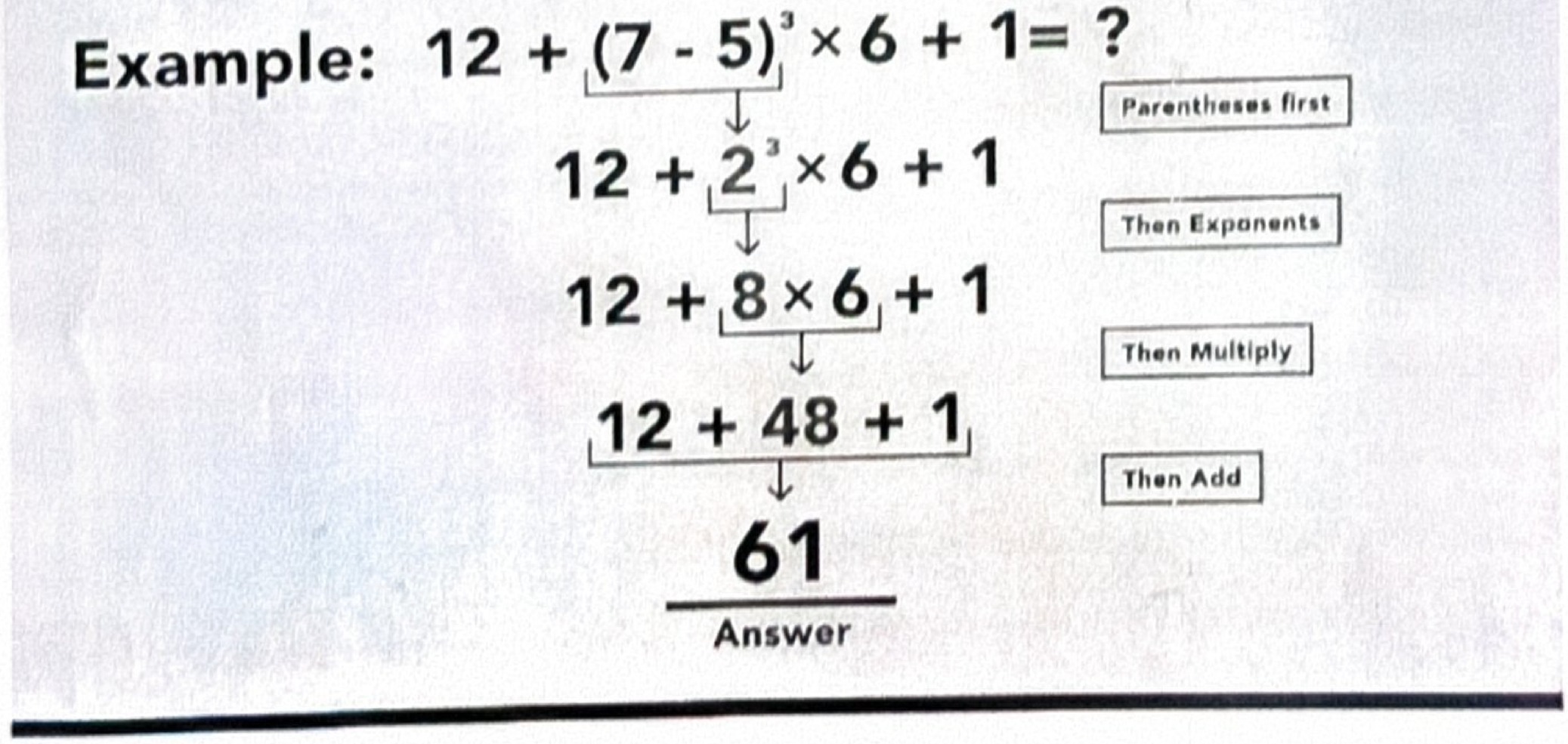
Just like multiplication and division, addition and subtraction are prioritized equally and are performed **from left to right** as they appear in the expression.

For example:

$10 - 4 + 3$

- First, we subtract: $10 - 4 = 6$, so we get $6 + 3$
- Then, we add: $6 + 3 = 9$
- Thus, the result is 9.

Now that we have covered each step of PEMDAS, let's look at a more challenging example:



COMMON MISTAKES WITH THE ORDER OF OPERATIONS IN MATH

PEMDAS makes solving math expressions much easier than simply memorizing the order of operations, but students sometimes forget the steps to take.

Here are some common mistakes to keep an eye on when solving math expressions with multiple operations:

1. Going Only Left to Right

Some students forget PEMDAS and just focus on the left-to-right approach, which might feel natural because of how we read.

It's important to remember that while going left to right can work, it's only suitable when you're solving expressions that only contain addition and subtraction, or multiplication and division.

For example, using the left to right approach works if you are trying to solve:

$8 - 6 + 9 - 3$

Or, if you are trying to solve:

$8 \div 2 \times 9 \div 3$

But let's say you are trying to solve:

$10 - 3 \times 2 + 5 \div 5$

As you can see, this expression contains subtraction, multiplication, addition, and division.

If we go from left to right – first subtracting 3 from 10 to get 7, then multiplying 7 by 2 to get 14, then adding 5 to it to get 19, and finally diving 19 by 5, we get 3.8 which is an incorrect result.

But if we follow PEMDAS, which tells us to start with multiplication and division (left to right) before we get to addition and subtraction, we get:

$10 - 3 \times 2 + 5 \div 5$

$= 10 - 6 + 1$

$= 5$

The **correct answer is 5**.

2. Doing Addition Before Subtraction

Many students tend to do addition before subtraction in math problems because they learned addition before subtraction or because they're close together in the order of operations. Some also think that because A comes before S in PEMDAS you should always do addition before subtraction.

For instance, in the expression $8 - 2 + 3$ you might tackle the addition before the subtraction and get: $8 - 5 = 3$.

In the order of operations, addition and subtraction have the same priority which is why we have to use left-to-right approach and perform subtraction first:

$8 - 2 + 3 \quad = 6 + 3 \quad = 9$

3. Doing Multiplication Before Division

Similarly to addition vs. subtractions, students often think multiplication comes before division because that's what they've learned before or because they're used to seeing multiplication listed first in math problems. Another mistake is believing M comes before D in PEMDAS because you always do multiplication before division.

For example, in the expression $6 \div 2 \times 3 = 6 \div 6 = 1$, a student might do the multiplication first and get a wrong result:

$6 \div 2 \times 3 = 6 \div 6 = 1$

But we know by now that multiplication and division are equally important. So, we need to work from left to right and do the division first:

$6 \div 2 \times 3 = 3 \times 3 = 9$

SOLVED EXAMPLES FOR THE ORDER OF OPERATIONS

Example 1

Using PEMDAS, let's solve this one:

$3 \times (8 - 4)^2 + 6 \div 2$

- First, we do the subtraction inside the parentheses: $8 - 4 = 4$, so we get $3 \times 4^2 + 6 \div 2$.
- Next, we square the result: $4^2 = 16$, so we get $3 \times 16 + 6 \div 2$.
- Then, we multiply: $3 \times 16 = 48$, so we get $48 + 6 \div 2$.
- Next, we divide: $6 \div 2 = 3$, so we get $48 + 3$.
- Finally, we add: $48 + 3 = 51$.

Example 2

Let's do the same for this expression:

$3 \times (7 - 2) + 6 \div 2$

- First, we do the subtraction inside the parentheses: $7 - 2 = 5$, so we get $3 \times 5 + 6 \div 2$.
- Next, we multiply: $3 \times 5 = 15$, so we get $15 + 6 \div 2$.
- Then, we divide: so we get $6 \div 2 = 3$, so we get $15 + 3$.
- Finally, we add: $15 + 3 = 18$.

Example 3

Finally, we can work on this one:

$80 \div (6 + 7 \times 2) - 5$

- First, we perform the multiplication inside the parentheses: $7 \times 2 = 14$, so we get $80 \div (6 + 14) - 5$.
- Next, we perform the addition inside the parentheses: $6 + 14 = 20$, so we get $80 \div 20 - 5$.
- Then, we divide: $80 \div 20 = 4$, so we get $4 - 5$.
- Finally, we subtract and get: $4 - 5 = -1$.

Algebraic Properties

- **Commutative Law:** The order of numbers doesn't change the result.
  - Addition:

    a+b=b+aa plus b equals b plus a

    $a+b=b+a$

  - Multiplication:

    a×b=b×aa cross b equals b cross a

    $a\times b=b\times a$

- **Associative Law:** The way numbers are grouped doesn't change the result.
  - Addition:

    (a+b)+c=a+(b+c)open paren a plus b close paren plus c equals a plus open paren b plus c close paren

    $(a+b)+c=a+(b+c)$

  - Multiplication:

    (a×b)×c=a×(b×c)open paren a cross b close paren cross c equals a cross open paren b cross c close paren

    $(a\times b)\times c=a\times(b\times c)$

- **Distributive Law:** Multiplication distributes over addition.

  a×(b+c)=(a×b)+(a×c)a cross open paren b plus c close paren equals open paren a cross b close paren plus open paren a cross c close paren

  $a\times(b+c)=(a\times b)+(a\times c)$

- **Identity Law:** A number remains unchanged when combined with the identity element.
  - Addition:

    a+0=a a plus 0 equals a

    $a+0=a$

  - Multiplication:

    a×1=a a cross 1 equals a

    $a\times 1=a$

- **Inverse Law:** A number can be combined with its inverse to result in the identity element.
  - Addition:

    a+(−a)=0a plus open paren negative a close paren equals 0

    $a+(-a)=0$

  - Multiplication:

    a×(1a)=1a cross open paren 1 over a end-fraction close paren equals 1

    $a\times(1a)=1$

Other Important Rules

- **Properties of Zero:** Any number plus zero equals itself, and any number multiplied by zero equals zero.
- **Properties of One:** Any number times one equals itself, and any number divided by one equals itself.
- **Equality:** If two things are equal to the same thing, they are equal to each other.