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Department of Mathematics
Analysis 3 (2^{nd} Year in Mathematics)

Series of Exercises 4

Exercise 1

Consider the function 2π-periodic f defined by:

$$f(x) = \frac{x^2}{2}, \quad \text{for } x \in [-\pi, \pi].$$

❶ Represent f.

❷ Show that f is expandable in Fourier series and compute its coefficients.

❸ By choosing particular values of x, deduce the sums of the following numerical series: $\sum_{n\geq 1} \frac{1}{n^2}$ and $\sum_{n\geq 1} \frac{(-1)^{n+1}}{n^2}$.

❹ Using Parseval's formula, calculate the sum $\sum_{n\geq 1} \frac{1}{n^4}$.

Exercise 2

Consider the function 2π-periodic f defined by:

$$f(x) = x, \qquad \text{for } x \in \,]-\pi, \pi[\,.$$

❶ Represent f.

❷ Show that f is expandable in Fourier series and compute its coefficients.

❸ By choosing particular value of x, deduce the sum of the numerical series $\sum_{n\geq 0} \frac{(-1)^n}{2n+1}$.

❹ Using Parseval's formula, calculate the sum $\sum_{n\geq 0} \frac{1}{(2n+1)^2}$.

Exercise 3 (Personal work)

Consider the 2π-periodic even function f defined by:

$$f(x) = x - \pi, \; x \in [0, \pi].$$

❶ Represent f.

❷ Prove that f is developable in a Fourier series and explain its sum on $[-\pi, \pi]$.

❸ Deduce the sum of the series $\sum_{n\geq 0} \frac{1}{(2n+1)^2}$ by choosing a particular value of x.

❹ Using Parseval's formula, calculate the sum $\sum_{n\geq 0} \frac{1}{(2n+1)^4}$.

Exercise 4 (Personal work)

Let α be a non-integer real number and let f be the $2\pi-$ periodic function defined on $\mathbb{R}$ by:

$$f(x) = \cos(\alpha x), \quad x \in [-\pi, \pi].$$

❶ Prove that f is developable in a Fourier series and determine this series.

❷ Is the Fourier series of f uniformly convergent on $\mathbb{R}$?

❸ Deduce the following identities:

$$\begin{aligned}
a)\ \frac{\pi}{\sin(\alpha\pi)} &= \frac{1}{\alpha} + 2\alpha \sum_{n\geq 0} \frac{(-1)^n}{\alpha^2 - n^2}. \\
\text{b) } \pi\cot(\alpha) &= \frac{1}{\alpha} + 2\alpha \sum_{n\geq 0} \frac{1}{\alpha^2 - n^2}. \\
\text{c) } \frac{\pi^2}{\sin^2(\alpha\pi)} &= \sum_{n\in\mathbb{Z}} \frac{1}{(\alpha - n)^2}.
\end{aligned}$$