University Abd elhafid boussouf
Mila

Institute of Science   **Departement of GM & EM**
and Technology

**Series N°4**
Forced vibrations of systemes
have one degree of freedom

Exercise 1

A mass *m* suspended from two springs can move only vertically.

The mass is subjected to a friction force proportional to the velocity

(*c* is the proportionality coefficient).

1. Compute the natural frequency w_0 of small oscillations.
2. Compute the frequency w_a of weakly damped small oscillations.

3- Under the condition $\dfrac{w_0 - w_a}{w_0} = \dfrac{1}{100}$, compute: the coefficient *c*,
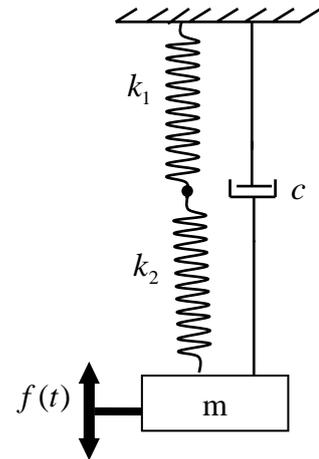
 the logarithmic decrement, and the quality factor Q.

4- A sinusoidal force $F(t) = F_0 \sin \omega t$ is applied to the mass *m*. Solve the differential equation

governing the motion.

Exercise

A mass (*m = 10 kg*) suspended from a spring ($k = 4000 Nm^{-1}$) is subjected to a harmonic

force of amplitude **400 N** and frequency $f = 1.6 Hz$;Determine:

1) The extension of the spring under the weight of *m*.

2) The static deflection δ_{st} of the mass under the effect of the maximum excitation force.

3) If at the initial instant ($x_0 = 0.1m, \dot{x}_0 = 10 ms^{-1}$), find the transient response and the

    steady-state response of the system to the excitation force.

Exercise 3

The system shown consists of a mass *m* connected to a fixed

support by a spring of stiffness *K* and, on the other side, by a
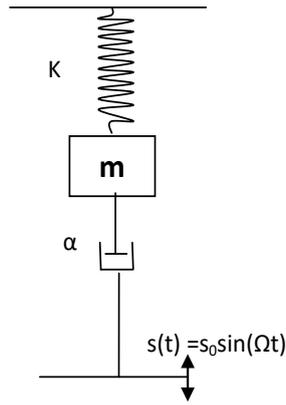
damper with viscous damping coefficient c.

The end of the damper moves according to $s(t) = s_O \sin(\Omega t)$.

1. Find the kinetic energy T and the potential energy U of the system.
2. Find the dissipation function.
3. Establish the differential equation of motion.
4. Determine the resonance frequency.

Exercise 4

A vehicle

$(m = 1200 kg, k_{eq} = 400 KNm^{-1}, \varepsilon = 0.5$

(where ξ is the damping ratio) moving on a
rough road at a speed of 20 km/h can be
modeled by the figure shown. If we
consider that the road surface varies
sinusoidally with an amplitude of 0.05 m
and a wavelength of 6 m, find:

$$y(t) = y_0 \sin wt$$
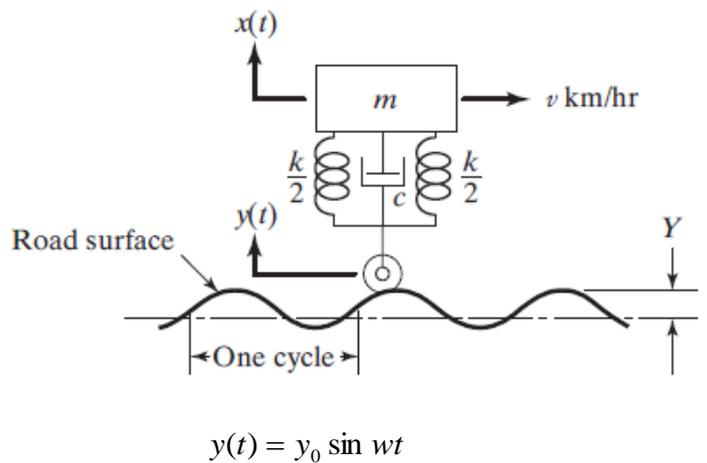
1. The natural frequency of the vehicle.
2. The value of the vehicle's damper.
3. The vertical displacement of the vehicle.
4. The ratio $\dfrac{x_{max}}{y_{max}}$ of the vehicle's maximum vertical displacement to Y.