Chapter 3

Statics

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I. Introduction

Statics is a branch of rational mechanics that deals with the equilibrium of material bodies with respect to a fixed frame of reference, as well as its methods for reducing a system of forces to an elementary form.

In this chapter, we attempt to define the four axioms of statics, as well as the different types of constraints (represented by their reactions) between the various parts of the rigid body and the reactions necessary to accurately determine the conditions for static equilibrium.

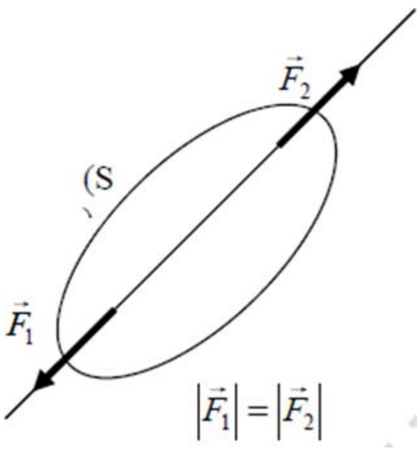
Throughout this discussion, we consider rigid bodies.

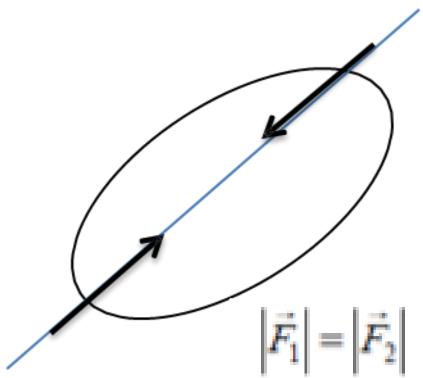
II. Axioms of statics:

1. First Axiom:

A rigid body (S) subjected to a system of two forces $\vec{f}1$ and $\vec{f}2$ is in equilibrium if:

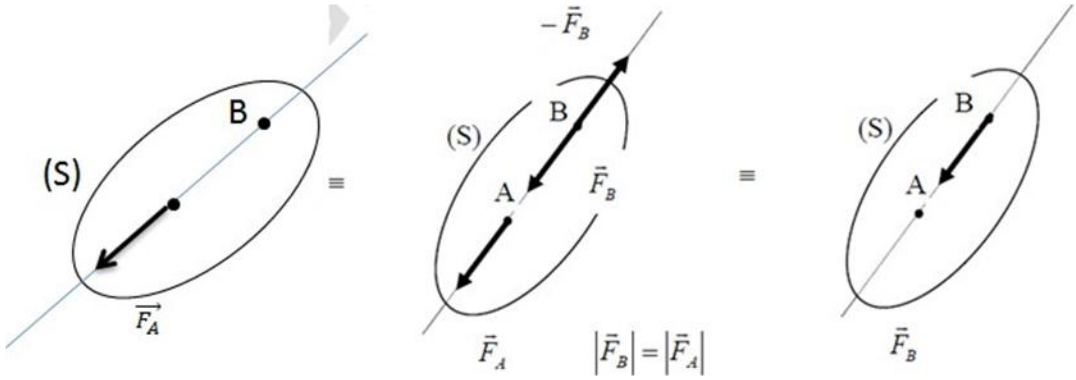
$\vec{f}1 = \vec{f}2$
same direction,
opposite senses





2. Second Axiom: (Sliding Principle)

The action of a given system of forces on a rigid body will not change if a balanced system of forces is added to or subtracted from that system.

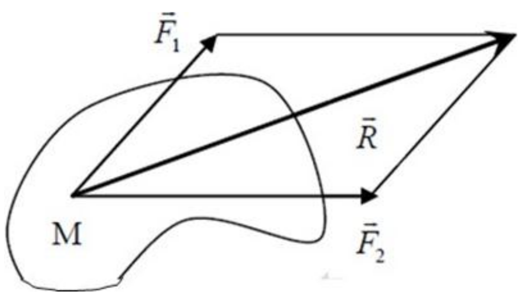


Principle of sliding: the point of application of a force can slide along its line of action within the rigid body.

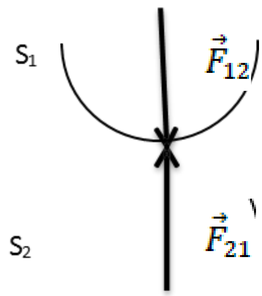
3. The 3rd axiom: (parallelogram principle)

Two forces acting at the same point on a rigid body form a parallelogram, whose diagonal represents the resultant of the two forces.

$\vec{R} = \vec{F_1} + \vec{F_2}$



4. The 4th axiom: (action-reaction principle)
Consider two solids (S1) and (S2) in contact with each other. If (S1) acts on (S2) with a force F→12, then (S2) will act on (S1) with a $\vec{F}_{21}$ e s such th $\vec{F}_{12} = -\vec{F}_{21}$



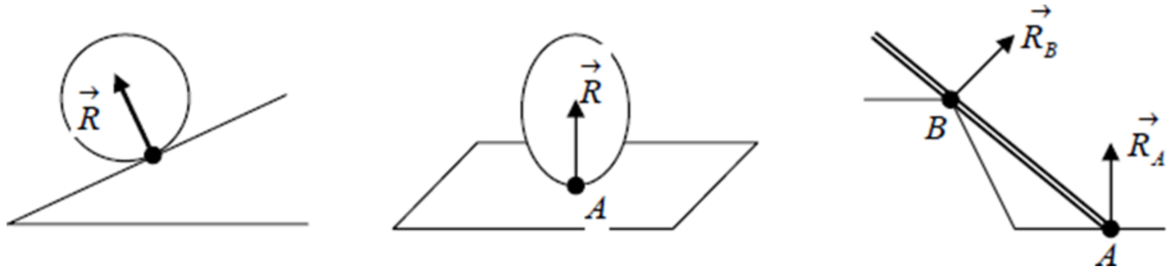
III. Constraints, Supports, and Reactions:

A body connected to or in contact with other bodies is limited in its movement. It is subject to constraints and is said to be hindered or not free.

Physically, a constraint is anything that limits the movement of a given body or allows it to be held in a specific position in space. A constraint is what is commonly referred to as a support.

1. Simple support of a solid on a perfectly smooth (frictionless) surface:

Let (S) be a solid resting on a surface (p). We say that point A of the solid is a support point if it remains continuously in contact with the surface "p".

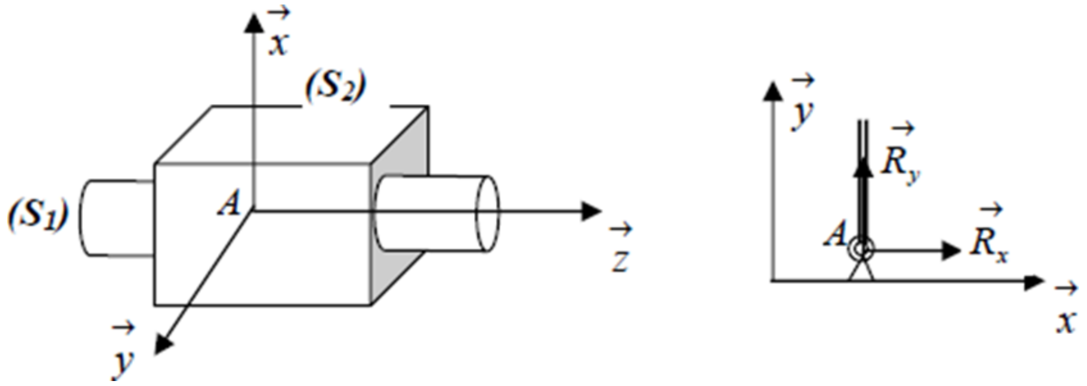


If the plane (P) is perfectly smooth, then the constraint force (the reaction $\vec{R}$) at the point of contact is normal to this plane.

2. Joints of a rigid body:
A point A of a rigid body is a joint when it remains permanently fixed at a point in space.

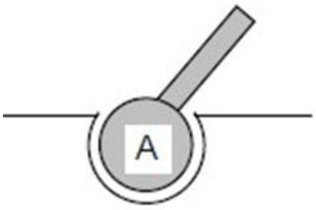
1. Lock joint (Cylindrical joint)

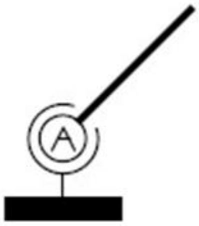
The rigid bodies are in contact with each other along a cylindrical surface. Rigid body (S1) has two degrees of freedom relative to rigid body (S2): a translation along the Az axis and a rotation about the same axis.

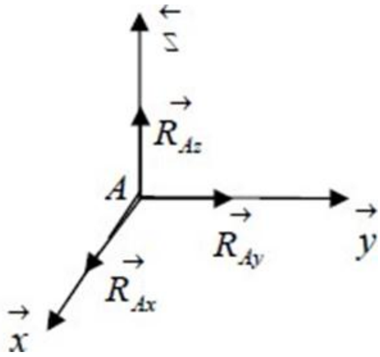


$\vec{R}_A = \vec{R}_{AX} + \vec{R}_{AY}$ with $\vec{R}_{AZ}=0$,he reaction along the axis of the joint (AZ) is zero.

2.2. Ball joint (Spherical joint):
The spherical joint has 3 degrees of freedom (rotation).





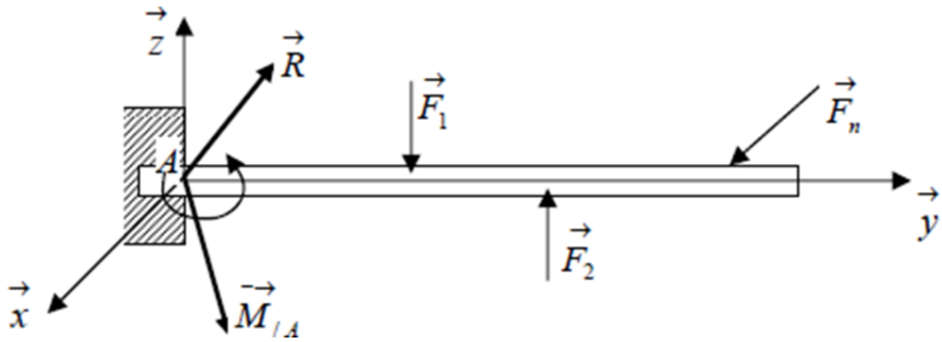


The reaction at point A of the spherical joint has three components

$$\overrightarrow{R_A} = \vec{R}_{AX} + \vec{R}_{AY} + \vec{R}_{AZ}$$

2.3. Fixed Support of a Rigid Body:

A rigid body is said to be fixed when its position cannot change regardless of the external forces applied. This constraint is represented by



$\vec{R}$:The resultant of the external forces applied to the rigid body and acting at point A

$M_{/A}$:The resultant moment of the external forces applied to the rigid body with respect to point A

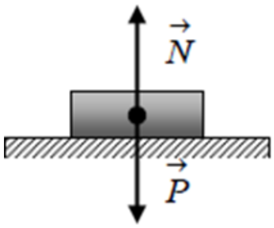
The following table summarizes the different bonds and their corresponding reactions.

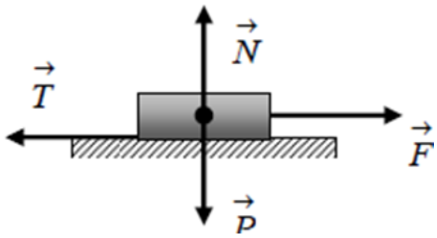
Types of connections	Components of the reaction
Simple support (roller) or smooth surface without friction	$\vec{R}$: the reaction is normal to the point of contact
Simple support with friction	$\overrightarrow{R_X}$, ,$\overrightarrow{R_Y}$:two components in the contact plane
Cylindrical joint with OZ axis	$\overrightarrow{R_X}$, $\overrightarrow{R_Y}$ With $\overrightarrow{R_Z}$=0: the component along the axis of the joint is zero
Spherical joint	$\overrightarrow{R_X}$, $\overrightarrow{R_Y}$, $\overrightarrow{R_Z}$: three components

Recessed	$\{\overrightarrow{R_X}, \overrightarrow{R_Y}, \overrightarrow{R_Z}$ and $\overrightarrow{M_{/A}}$ three components plus the moment at the points of the embedment

3. Connections between solids with friction:

A rectangular piece of wood is placed on a horizontal plane. This piece is in static equilibrium; the reaction force of the plane is equal and opposite to the weight of the piece.



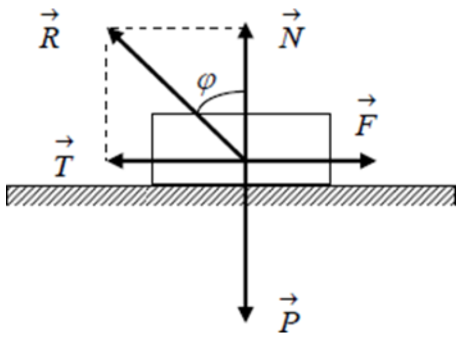


By gradually applying a horizontal force $\vec{F}$,to a point on this part, the part will not move as long as this force is less than a certain limit. There is then a counterforce $\vec{T}$that balances and opposes $\vec{F}$; $\vec{T}$ is called the static friction force.

In the case of a surface with friction, the equilibrium condition is written

$$\vec{N} + \vec{T} + \sum \vec{F}_i = \vec{0} \Rightarrow tg\varphi = \frac{T}{N} = f$$

f is the coefficient of friction



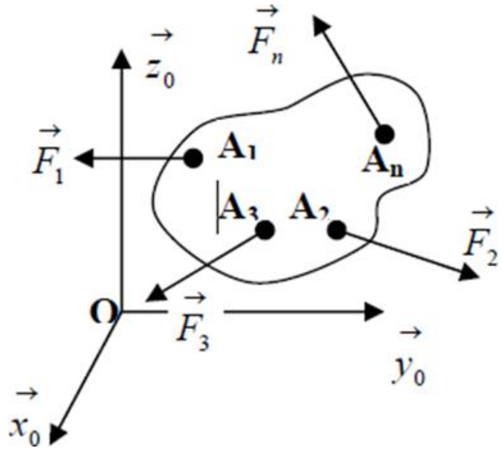
IV. Conditions for equilibrium:

⟶ **Equilibrium of the rigid body in space**

For the rigid body to be in static equilibrium, it is necessary and sufficient that:

-The resultant of all external forces applied to the rigid body is zero;

-The resultant moment of all these forces at a point O is zero.



A rigid body (S), subjected to external mechanical actions, is in static equilibrium if and only if the wrench representing all these actions is a zero wrench.

These two vector equations translate into the following six scalar equations.

$$\vec{R} = \vec{0} \Rightarrow \begin{cases} R_x = 0 \\ R_y = 0 \\ R_z = 0 \end{cases} \quad \text{et} \quad \vec{M}_{/0} = \vec{0} \Rightarrow \begin{cases} M_x = 0 \\ M_y = 0 \\ M_z = 0 \end{cases}$$

The system is completely determined if the number of unknowns is equal to the number of independent equations.

Equilibre d'un solide dans un plan

Dans le cas d'un solide soumis à des forces coplanaires, le système précédent se réduit à trois équations scalaires.
Soit (xoy), le plan contenant les forces appliquées au solide, nous avons alors : Z=0 et Z=0 et F_Z=0 ➡ M_X=M_Y
Les conditions d'équilibre alors se réduisent à 3 équations scalaires

The equilibrium conditions then reduce to 3 scalar equations:

$$F_x = \sum F_{ix} = 0 \qquad F_y = \sum F_{iy} = 0, \qquad M_z = \sum M_{iz} = 0$$

$$\vec{F}_i = \begin{cases} F_{ix} \\ F_{iy} \\ 0 \end{cases} \quad ; \quad \overrightarrow{OA}_i = \begin{cases} x_i \\ y_i \\ 0 \end{cases}$$

$$\overrightarrow{M}_{i/o} = \overrightarrow{OA}_i \wedge \vec{F}_i = \begin{cases} x_i \\ y_i \\ 0 \end{cases} \wedge \begin{cases} F_{ix} \\ F_{iy} \\ 0 \end{cases} = \begin{cases} 0 \\ 0 \\ x_i F_{iy} - y_i F_{ix} = M_{iz} \end{cases}$$