Chapter 3: Electrical circuit and power

1. Example of a single-phase electrical installation study

A single-phase electrical installation supplied with a voltage of 230 V; 50 Hz includes:

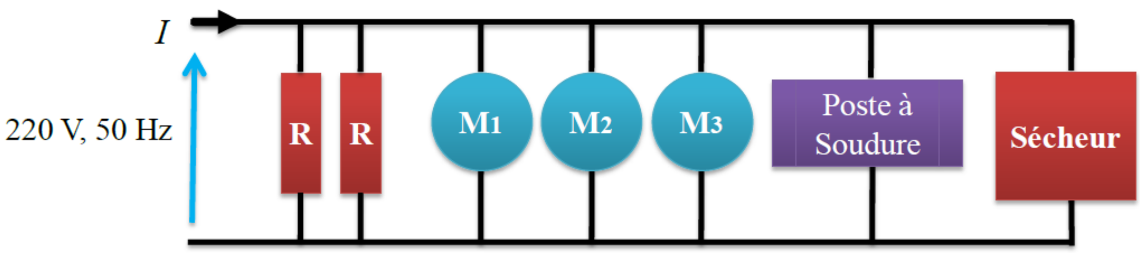
- Two radiators with a power rating of P = 5 kW each.
- Three motors, each absorbing 3 kW, with the following power factors:
  PF1 = 0.74
  PF2 = 0.72
  PF3 = 0.71
- A welding station with an electrical power of P = 4 kW and a power factor of PF = 0.83.
- An electric dryer absorbing 5 kW.

The objective is to determine the power balance of the installation. For this purpose, we will:

1. Calculate the total active power (P) when all loads are operating.
2. Calculate the total reactive power (Q) when all loads are operating.
3. Calculate the total apparent power (S), and then determine the overall power factor (PF) of the installation as well as the line current (I).
4. Add a capacitor in parallel with the installation. Determine the required capacitance value to improve the power factor to 0.94.

Solution:

We start with a graphical representation of the installation, and then we determine the power of each element of the installation.



1- Calculation of the total active power (P)

To calculate this power, we first determine the **active power of each element**, then apply **Boucherot's theorem**. For the **radiators**, the absorbed power is entirely **dissipated as heat**, therefore the **reactive power is zero**.
Mathematically, for a **purely resistive load**, the **phase shift** is zero ($\Delta\varphi = 0$). Thus:

Two radiators : $P_{2R} = 2*5 = 10$ KW
One welding station : $P_{ws} = 4$ KW

One electric dryer : P_{ed} = 5 kW.

Three motors :

$P_{3M} = P_{M1} + P_{M2} + P_{M3}$

$=3 * P$

P3M = 9KW

$$\boldsymbol{Pt} = P2R + Pws + Ped + P3M = 10 + 4 + 5 + 9 = 10 + 4 + 5 + 9 = 28\ KW$$

2- Calculation of the Total Reactive Power (Q)

The reactive power, according to the power triangle, is given by:

$$Q = P \times tang(\varphi)$$

Two radiators :$Q_{2R} = P \times tang(0) = 0\ VAR$

One welding station : $Q_{ws} = P \times tang(\varphi)$

$$\varphi = cos^{-1}(0.83) = 33.90°$$

$$Q_{ws} = 4 \times tang(33.90°) = 2.68\ KVAR$$

One electric dryer : $Q_{ws} = 0\ VAR$

Three motors : $Q_{3M} = Q_1 + Q_2 + Q_3$

$Q_1 = P_1 * \tan\varphi_1$	$Q_2 = P_2 * \tan\varphi_2$	$Q_3 = P_3 * \tan\varphi_3$
$\varphi_1 = \cos^{-1}(0.74)$	$\varphi_2 = \cos^{-1}(0.72)$	$\varphi_3 = \cos^{-1}(0.71)$
$\varphi_1 = 42.26°$	$\varphi_2 = 43.94°$	$\varphi_3 = 44.76\ °$
$Q_1 = 3 * \tan(42.26)$	$Q_2 = 3 * \tan(43.94)$	$Q_3 = 3 * \tan(44.76\ °)$
$Q_1 = 2.73$ kVAR	$Q_2 = 2.89$ KVAR	$Q_3 = 2.97$ KVAR

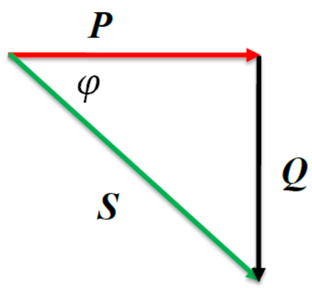
Thus: Q_{3M} = 2.73 + 2.89 + 2.97 =8.60 KVAR

$$\boldsymbol{Q_t} = Q_{2R} + Q_{ws} + Q_{ed} + Q_{3M} = 0 + 2.68 + 0 + 8.60 = 11.28\ KVAR$$

3- Then calculate the total apparent power S, and deduce the power factor (Fp) of the installation, as well as the line current (I).

$$S = \sqrt{(P_T)^2 + (Q_T)^2} = \sqrt{(28)^2 + (11.28)^2}$$

$$\boldsymbol{S = 30.18\ KVA}$$



- The power factor

$$\cos\varphi = \frac{P}{S} = \frac{28}{30.64} = 0.91$$

$$\mathbf{\cos\varphi = 0.91}$$

- The line current (I)

We have the equation : S = V. I, So :

$$I = \frac{S}{V} = \frac{30.64 * 10^3}{220} = 139.27\ A$$

$$\boldsymbol{I = 139.27\ A}$$

- **Improvement of the Power Factor**

In industrial applications, it is often desirable that the apparent power **S** be as close as possible to the active power **P**, meaning that the phase angle tends toward zero (0°) and the power factor toward 1.
In the common case of motors (inductive loads), it is very often possible to improve the power factor by connecting capacitors in parallel with the load.
Across the terminals of the capacitor, the phase shift is negative: $\Delta\varphi = -\pi/2$

The active power of the capacitor is:

$$P_c = VI\cos\left(-\frac{\pi}{2}\right) = VI * 0$$

$$\boldsymbol{P_c = 0\ W}$$

It should be noted that the active power does not change.
The reactive power is:

$$Q_c = VI\sin\left(-\frac{\pi}{2}\right)$$

$$Q_c = -V^2\omega C$$

Since the reactive power of the capacitor is negative, then:

$$\acute{Q}_T = Q_T - Q_c$$

$$\acute{Q}_T < Q_T$$

If we want to improve the power factor of the previous installation to **0.94**,

$$P'_T = P_T$$

Reactive power:

$$Q'_T = P'_T \tan\varphi'$$

$$\cos\varphi' = 0.94$$

$$\varphi' = 19.94°$$

$$\tan\varphi' = 0.36$$

$$Q'_T = P'_T \tan\varphi' = 28 * 0.36 \text{ KVAR}$$

$$Q'_T = 10.16 \text{ KVAR}$$

$$\Delta Q = Q'_T - Q_T = 10.16 - 11.28$$

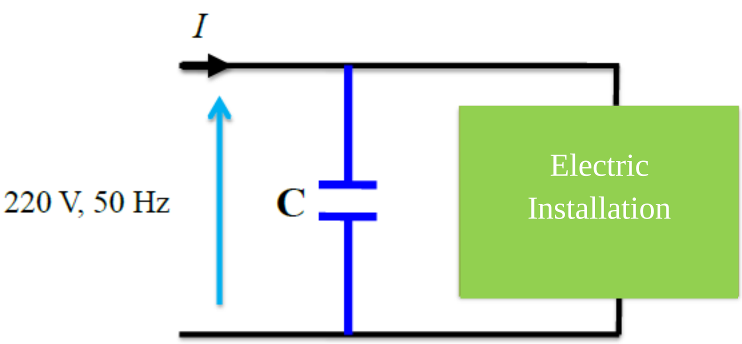
$$\Delta Q = Q_C = -1637.35 \text{ VAR}$$

$$-1637.35 = -V^2\omega C$$

$$C = \frac{\Delta Q}{V^2 2\pi F} = \frac{1637.35}{(220)^2 * 2\pi * 50} = \frac{1637.35}{16610600} = 107.68\ \mu F$$

$$C = 107.68\ \mu F$$

A capacitor is then connected in parallel with the installation:



It is very interesting to note that the active or useful power **P** does not change, since the voltage across the load remains the same. However, the reactive power supplied by the capacitor is negative, which means that the total reactive power decreases and the power factor increases. As a result, both the current and the apparent power decrease, leading to an improvement in the efficiency of the distribution system.

2. Three-phase system

In industry, single-phase power or a single-phase network is generally insufficient. The use of three-phase networks makes it possible to triple the available power.
This network consists of three sinusoidal alternating currents of the same frequency and amplitude (see Figure 1), but phase-shifted from each other by 120°

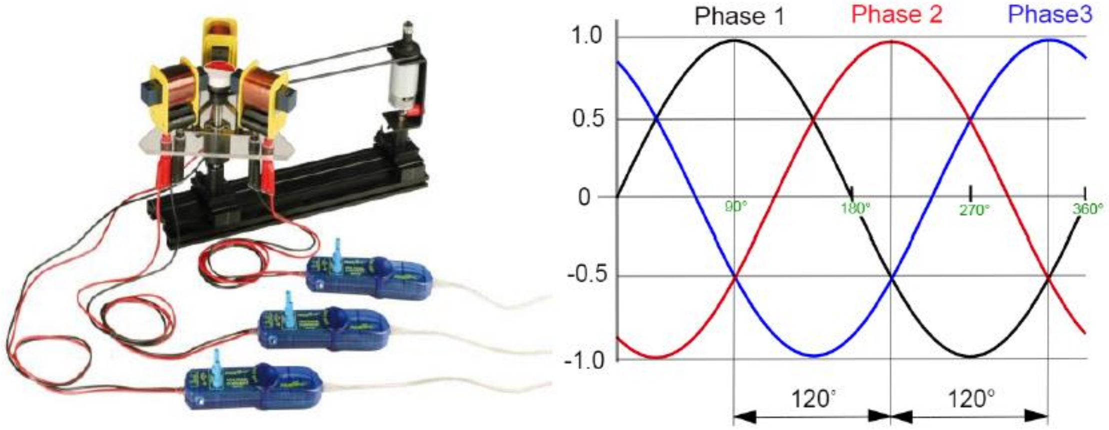


Figure 1. Three-phase system.

This network is generated by a three-phase synchronous machine composed of a **rotor** (rotating magnet) and **three fixed windings**.
The windings are spaced 120° apart around the rotor. Since they are physically separated by 120°, the voltages generated in the windings V_1, V_2 and V_3 are phase-shifted by 120°.

$$\begin{cases} V_1(t) = V_M \sin(\omega t) \\ V_2(t) = V_M \sin(\omega t + 120°) \\ V_2(t) = V_M \sin(\omega t - 120°) \end{cases}$$

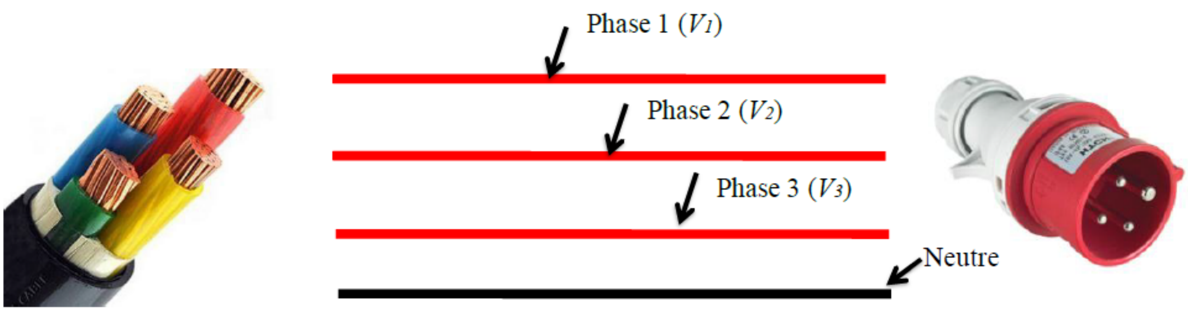


Figure 2. Three-phase socket.

2.1.Concepts and Definitions

Subscribers can choose to receive one of the following configurations:

- **Four conductors:** three phase conductors and one neutral conductor, which constitutes a **three-phase distribution**.
  or
- **Two conductors:** one phase conductor and one neutral conductor.

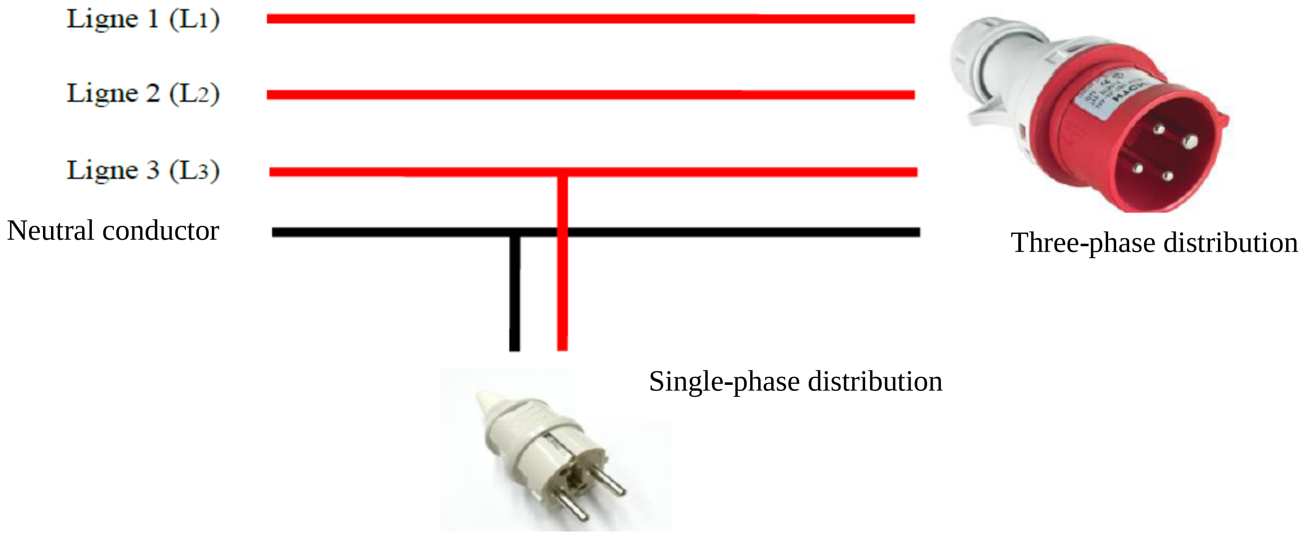


Figure 3. Mixed distribution.

2.2.Line-to-neutral and Line-to-line Voltages

<u>The **line-to-neutral voltages**</u> (also called **phase voltages**) are the voltages V_1, V_2 and V_3 that can be measured between the neutral point and each of the three phases or lines (L_1, L_2, and L_3). The three measured voltages are equal and have an effective value of **230 V** in domestic distribution systems.

Figure 4 shows the three line-to-neutral voltages.

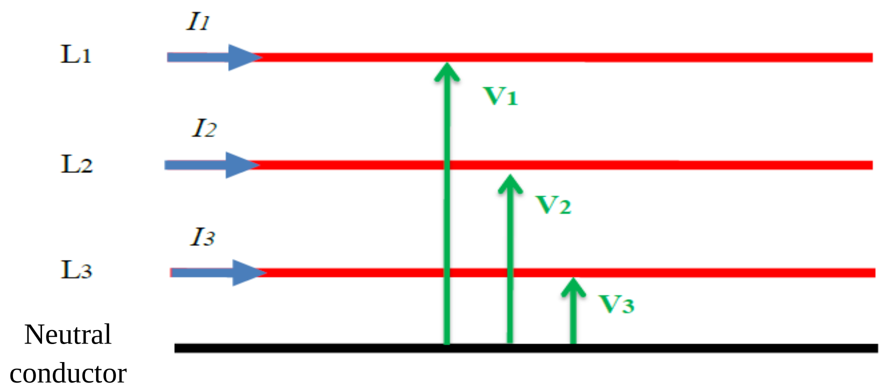


Figure 4. Single-phase voltages (Line-to-neutral voltages).

Line-to-line Voltages

The **line-to-line voltages** are the voltages U_{12}, U_{23} and U_{13}, which can be measured between phases (respectively between terminals 1 and 2, 2 and 3, and 3 and 1).
The three measured voltages are equal and have an effective value of **380 V**. Figure 5 shows the three line-to-line voltages.

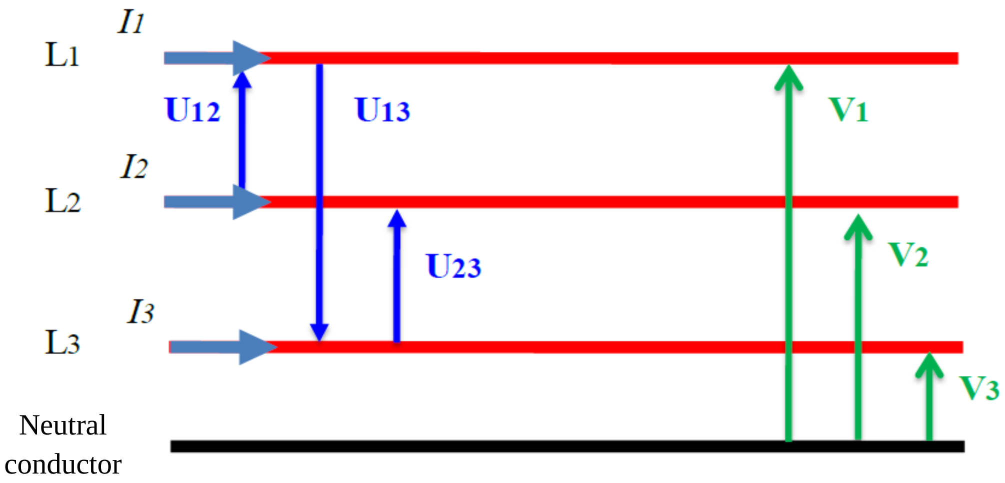


Figure 5. Line-to-Line voltages.

2.3.The relationship between U and V

We observe that $U12 = V1 - V2$ the same remark applies to the other line-to-line voltages. The vector representation is:

$$\begin{cases} \vec{U}_{12} = \vec{V}_1 - \vec{V}_2 \\ \vec{U}_{23} = \vec{V}_2 - \vec{V}_3 \\ \vec{U}_{13} = \vec{V}_1 - \vec{V}_3 \end{cases}$$

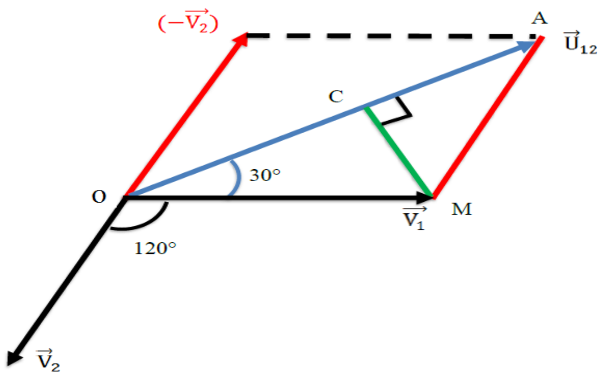
We can write for U_{12} :

$$\vec{U}_{12} = \vec{V}_1 - \vec{V}_2 = \vec{V}_1 + (-\vec{V}_2).$$

The vector of U_{12} is given by

Vector representation of the phase

voltages:



In triangle OAM, $OM(\overrightarrow{V_1})$ is equal to $MA\ (\overrightarrow{-V_2})$ therefore, it is an isosceles triangle. The height MC divides the base OA into two equal segments, OC and CA.

Therefore, the angle $\widehat{AOM}$ is equal to 30° with :

$$\cos 30° = \frac{OC}{OM}$$

$$OC = OM \cos 30° = OM\frac{\sqrt{3}}{2}$$

$$U_{12} = OA = 2\,OC = 2\,OM\frac{\sqrt{3}}{2}$$

$$U_{12} = OM\sqrt{3}$$

$$U_{12} = V_1\sqrt{3}$$

In general, we have the following relation: $\boldsymbol{U = V\sqrt{3}}$

2.4.Unbalanced three-phase system without neutral

In an unbalanced three-phase system without a neutral wire, the line-to-line voltages (the compound voltages) remain constant, but the phase voltages do not.

The current in the line is denoted by **I**,
and the current flowing through the load is denoted by **J**.

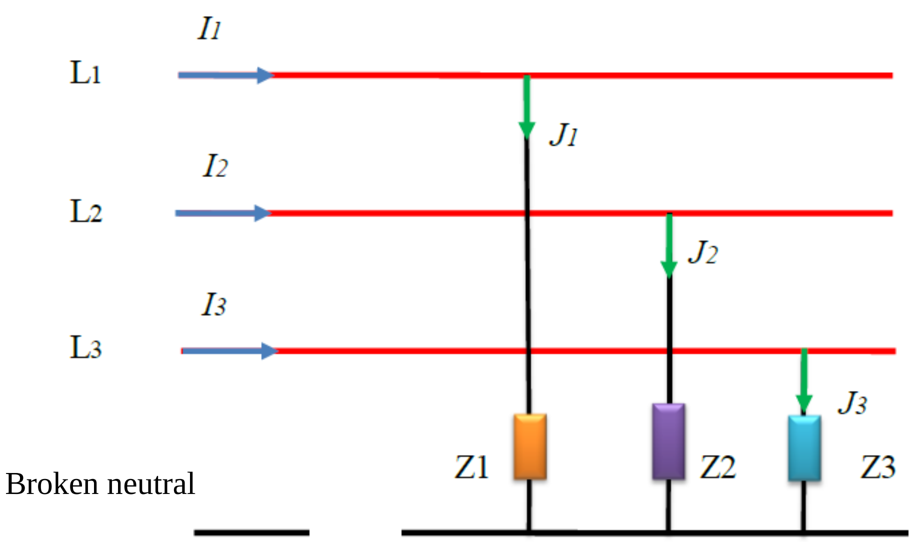


Figure 6. Unbalanced three-phase system.

2.5.Electric power in a balanced system

In a balanced three-phase system, the vector sum of the currents is zero.

$$\vec{I}_1 + \vec{I}_2 + \vec{I}_3 = \vec{0}$$

Two types of coupling (phase connection) can be distinguished at the receiver side:

– Star connection.

– Delta connection.

2.6.Balanced star connection

The star connection of loads in a three-phase system consists of forming a three-branch star using the three phases. The three loads have the same impedance $Z1 = Z2 = Z3$, which means that the current flowing through each load connected in a star is the same as the current coming from the corresponding phase, $I = J$.

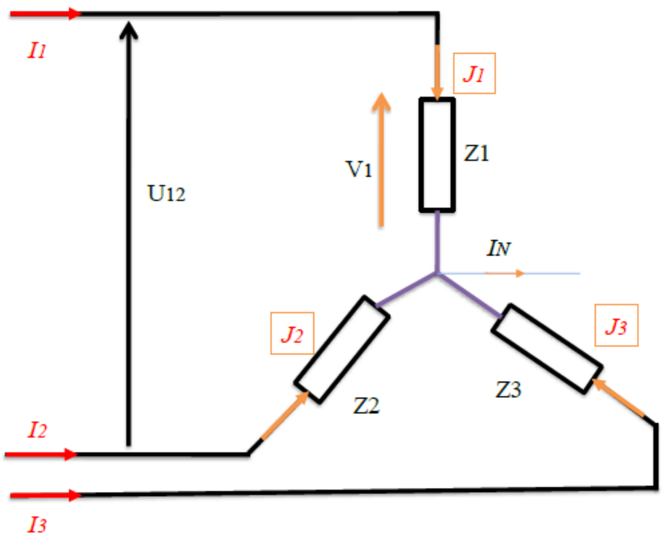


Figure 7. Star connection.

$$P_T = P_1 + P_2 + P_3 = 3\ P_{mono}$$
$$P_T = 3\ VI * \cos\varphi = 3\ VJ * \cos\varphi$$

P_{mono} : represents the power of a single phase.

$$V = \frac{U}{\sqrt{3}}$$

$$P_T = 3\,\frac{U}{\sqrt{3}} I * \cos\varphi = \sqrt{3}\,UI\cos\varphi$$

$$P_T = \sqrt{3}\,UI\cos\varphi$$

The reactive power Q_T is given by:

$$Q_T = \sqrt{3}\,UI\sin\varphi$$

The apparent power S will then be:

$$S = 3\,VI = \sqrt{3}\,UI = \sqrt{{Q_T}^2 + {P_T}^2}$$

2.7.Balanced delta connection

In a delta connection, the load is traversed by the line current J and subjected to the line-to-line voltage U. It should be noted that there is no neutral in the delta connection.

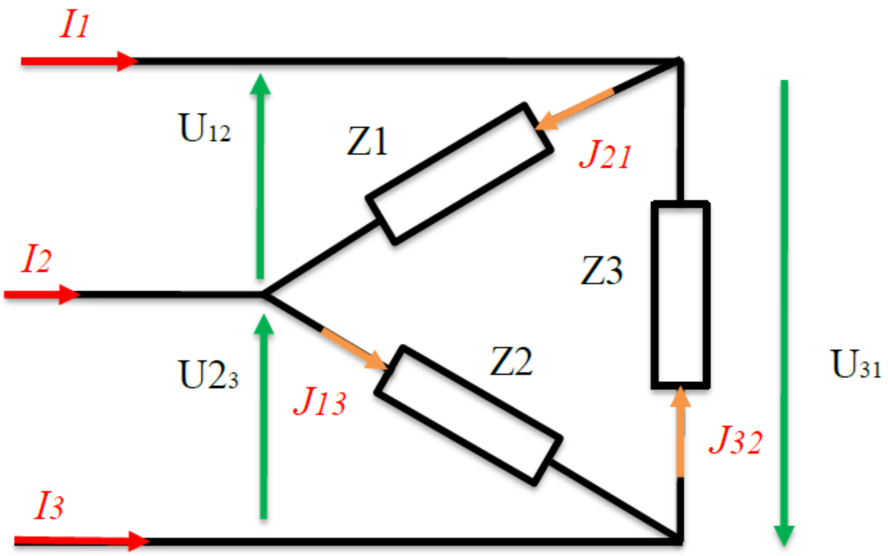


Figure 8. Delta connection**.**

For a load : $P_{mono} = UJ * cos\varphi$

For the network (3 loads) :

$$P_T = 3UJ * \cos\varphi$$

$I = J\sqrt{3}$, $J = \frac{I}{\sqrt{3}}$ alors :

$$P_T = \sqrt{3}\,UI * \cos\varphi$$

The reactive power is:

$$Q_T = \sqrt{3}\,UI \sin\varphi$$

Application

Consider a balanced three-phase load made up of three resistive loads R=100 Ω. This load is fed by a 220 V/380 V, 50 Hz supply.

1. **Star (Y) connection.**
   Calculate the rms value I of the line current and then the active (real) power.
2. **Delta (Δ) connection.**
   Calculate the rms value I of the line current and then the active (real) power.

What can be concluded?

Solution

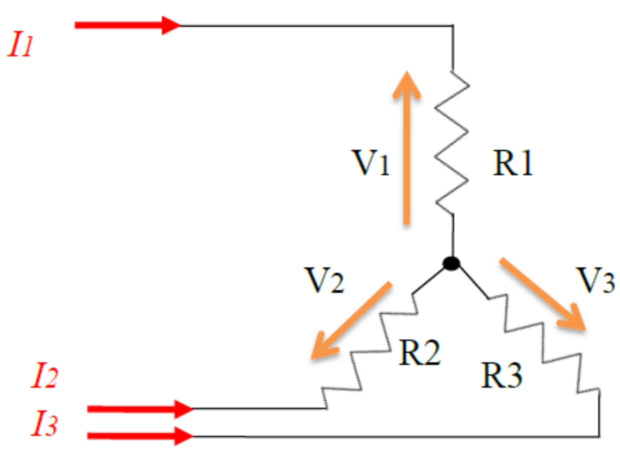
1- In the star connection, a balanced network : $I1 = I2 = I3 = I$

At the terminals of the load, we have:

V= RI, I=V/R

I=220/100= 2.2 A.

$$I = \frac{V}{R} = \frac{U}{\sqrt{3}R} = \frac{380}{\sqrt{3}\,100} = 2.2\ A$$



$$I_{star} = 2.2\ A$$

Symmetrical load: $P_T = 3\ VI = 3\ RI\ I = 3\ RI^2$

$$P_T = 3 * 100 * 2.2^2 \approx 1.45\ KW$$

$$P_{star} = 1.45\ KW$$

2- In the delta connection, the load is traversed by the line current J and subjected to the line-to-line voltage U; therefore:

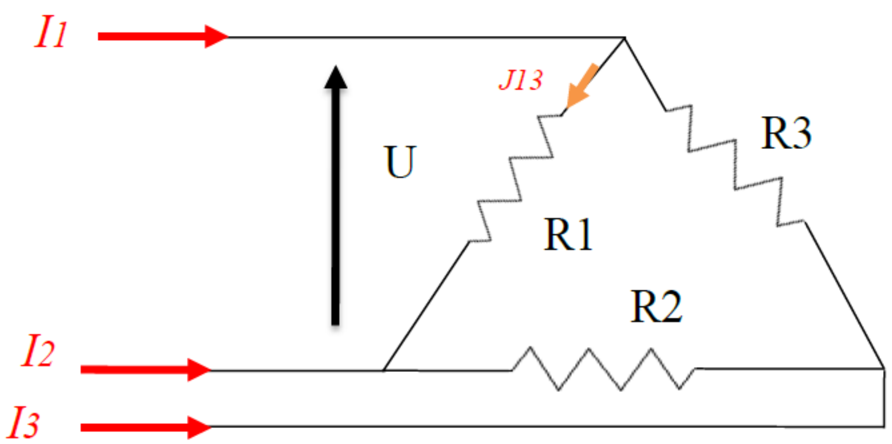
$$U = J * R, J = \frac{U}{R}$$

$$J = \frac{380}{100} = 3.8\ A$$

$$J = 3.8\ A$$

$$I = \sqrt{3}J = \sqrt{3} * 3.8 \approx 6.6\ A$$

$$I_{delta} = 6.6\ A$$



$$P_T = 3RJ^2 = 3 * 100 * 3.8^2 = 4.33\ KW$$

$$P_{delta} = 4.33\ KW \approx 3 * 1.43\ KW$$

3- Conclusion

It can be observed that the line current in a delta connection is three times greater than that in a star connection.

The same observation applies to the power: in the delta connection, the power is much higher and is three times greater than that of the star connection.

In industry, both types of connections are used for motor starting. This technique is called **star–delta starting**.