### 3.1. Introduction

Tension or compression corresponds to forces acting perpendicularly to the cross-sections of the parts. It is called uniaxial because the sides of the part are not constrained, and all the forces act along the same axis.

### 3.2. Definitions

Let a straight bar be subjected to two equal and directly opposite forces acting along its central axis, it is then subjected to a normal force (Fig. 3.1). This force is called:

- a simple tension force if the forces tend to elongate the bar;
- a simple compression force if the forces tend to shorten the bar.

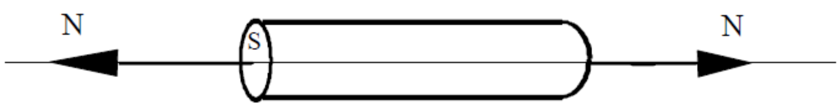



**Fig 3.1**. Bar under tension.

### 3.3. Normal Stress

Consider a straight bar with a cross-sectional area S, fixed at its upper end to a rigid support (Fig. 3.2-a). At the other end, it is subjected to the action of a force N applied along its axis.

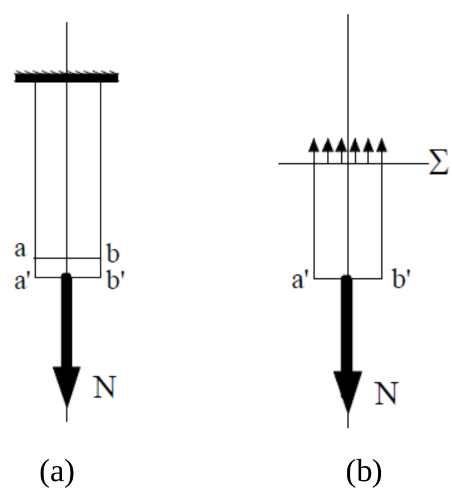



**Fig 3.2**. Fixed bar subjected to tension.

According to the principle of action and reaction, the support exerts a reaction force equal and opposite to N. The bar is thus subjected to a normal force. Its base ab then moves parallel to itself to the new position a'b'.

All the fibers experience, if the force is a tensile force, the same elongation (according to the Navier-Bernoulli hypothesis, which states that cross-sections remain plane and perpendicular to the axis) and therefore sustain the same tension.

If we imagine cutting the bar by a plane Σ perpendicular to its axis, in order to keep the lower portion in equilibrium, an internal force equal and opposite to N must be applied within Σ.

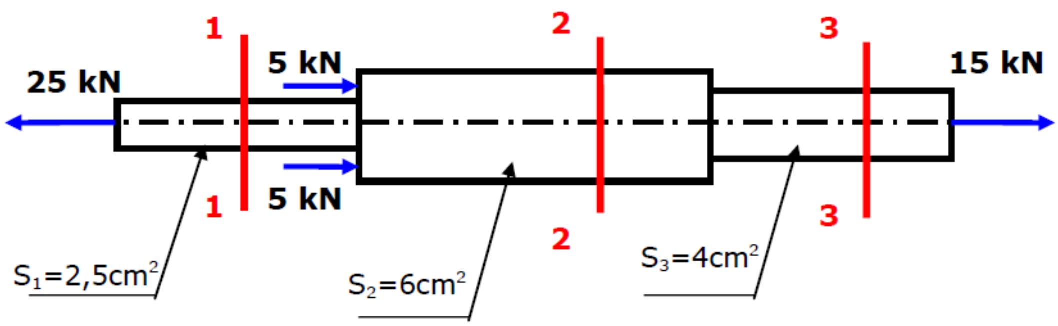
The Navier-Bernoulli hypothesis then allows us to write:

$$\sigma = \frac{N}{S} \qquad 3.1$$

σ is called the normal stress. It represents the intensity of the normal force per unit area. σ is measured in ($N/m^2$) or (Pa).

**Example 3.1**

Consider the bar shown in the figure below. Calculate the stresses at sections 1-1**,** 2-2 and 3-3.

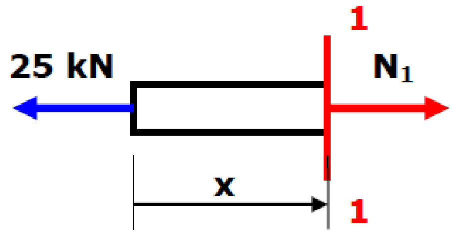



**Solution 2.1**

Section 1-1

$$\sum F_x = 0 \implies N_1 = 25\,KN$$

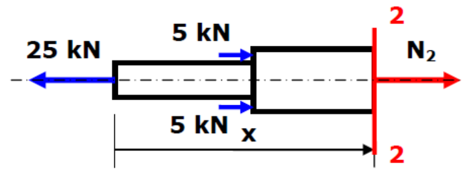
$$\sigma_{1-1} = \frac{N_1}{S_1} = \frac{25}{2.5} = 10 KN/cm^2 = 100\,MPa$$



Section 2-2

$$\sum F_x = 0 \implies N_2 = 15\ KN$$

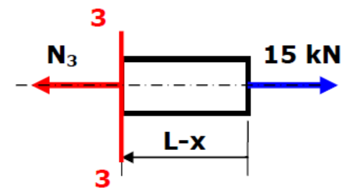
$$\sigma_{2-2} = \frac{N_2}{S_2} = \frac{15}{6} = 2.5\ KN/cm^2 = 25\ MPa$$




Section 3-3

$$\sum F_x = 0 \implies N_3 = 15\ KN$$

$$\sigma_{3-3} = \frac{N_3}{S_3} = \frac{15}{4} = 3.75\ KN/cm^2 = 37.5\ MPa$$

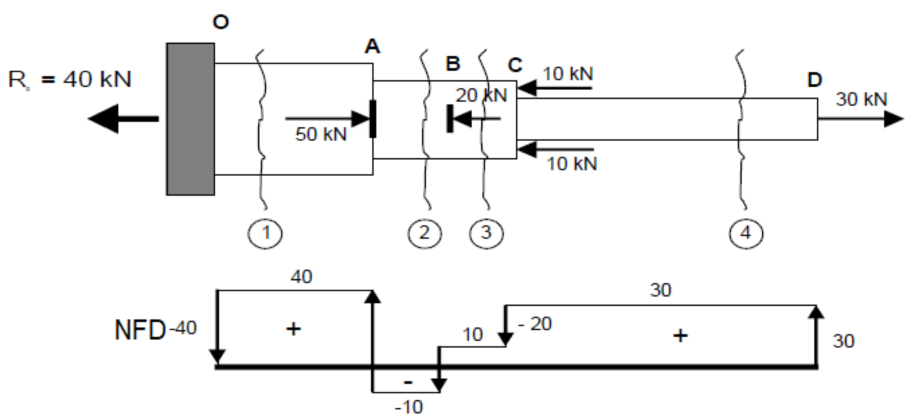



### 3.4. Normal Force Diagram

- The Normal Force Diagram shows the value of the normal force in all the cross-sections perpendicular to the member under study.
- The normal force in a section is the resultant of the axial loads acting on that section.
- The Normal Force Diagram is obtained using the method of sections, by making cuts at the points where each concentrated load is applied, as well as at the beginning and end, and at the minimum and maximum points (if any) of each distributed load.

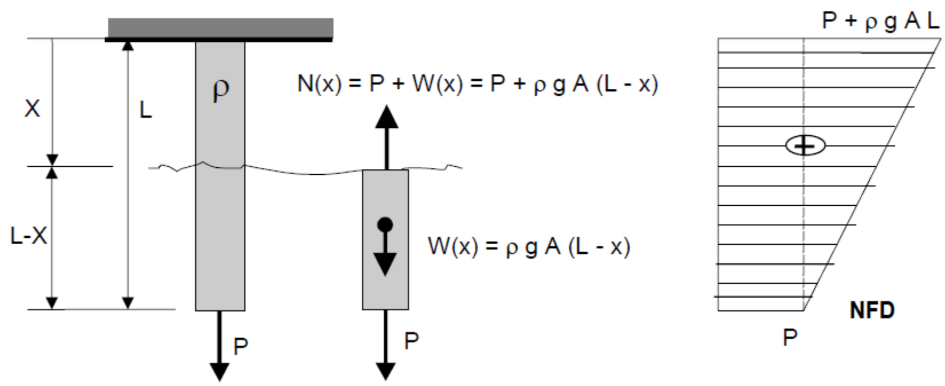
❖ **Example with Concentrated Forces**

The figure below illustrates the Normal Force Diagram along a bar in the case where the axial forces are concentrated.



### ❖ Example with a Distributed Load

The figure below illustrates the Normal Force Diagram along a bar subjected to its own weight.




### 3.5. Stress – Strain Curve

The stress–strain curve characterizes the behavior of a material. It is obtained experimentally from a tensile test performed on a bar with a constant cross-section. During this test, the normal force is gradually increased, causing the bar to elongate.

At each load increment, the normal stress and the corresponding strain of the bar are plotted on a graph. This process continues systematically until the bar fractures.

The resulting graph is the stress–strain curve of the material. In general (in a simplified form), it has the shape shown in Figure 3.3.

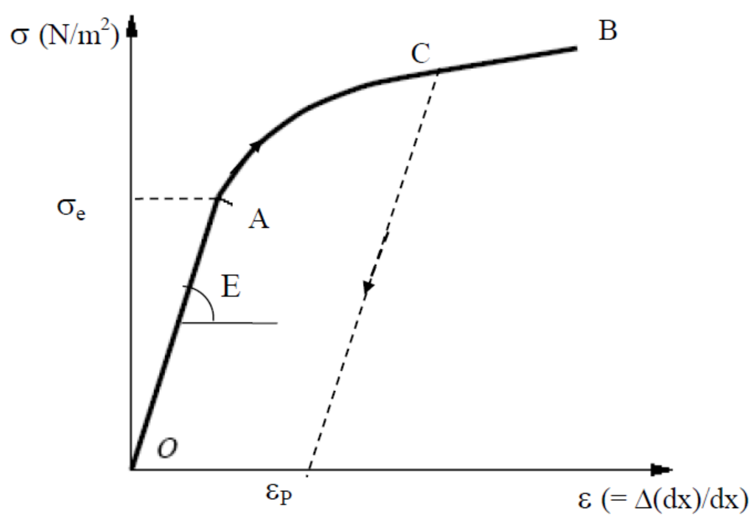



**Fig 3.3.** Tensile Stress – Strain Curve

The segment (OA) represents the elastic region. The elastic limit has not yet been reached. The bar returns to its original shape if the experiment is stopped within this region.

In this case, the elasticity is linear, and (OA) is a straight line. The slope E of this line is called the linear modulus of elasticity or Young's modulus (see Table 2.1). It represents the ratio between the stress and the strain (ε) in the elastic region.

Hooke's Law gives the relationship between stress and strain in the elastic region:

$$\sigma = E.\varepsilon \qquad 3.2$$

The segment (AB) represents the plastic region. The elastic limit has been exceeded. If the experiment is stopped (at point C), the bar does not return to its original shape.

The unloading path is, in a simplified way, parallel to the line (OA). When the applied force is completely removed, a residual strain ($\varepsilon_p$) remains, which persists even after the force is removed.

**Table 3.1.**Typical Values of Young's Modulus (E)

| **Material** | **steel** | **concrete** | **Aluminum** |
|---|---|---|---|
| **E ($daN/mm^2$)** | 21000 | 2000 | 7000 |

### 3.6. Condition of Strength

To verify a component's strength condition under tension or compression, one must ensure that:

$$\sigma \leq [\sigma] \qquad 3.3$$

Where [σ] is the allowable (or permissible) stress for the material under study. It is given by the expression:

$$[\sigma] = \frac{\sigma_e}{n} \qquad 3.4$$

Where $\sigma_e$ is the elastic (tensile) limit, and n is a safety factor (n > 1).

- **Elastic Limit**

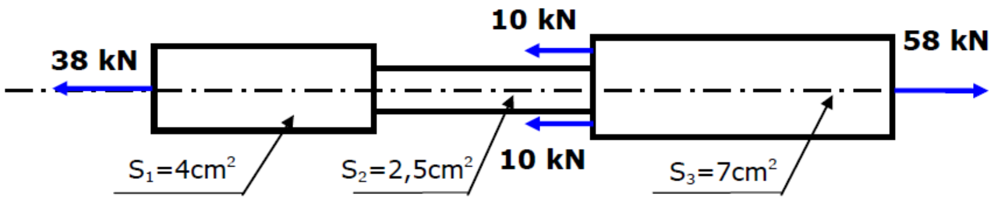
For all homogeneous and isotropic materials, the elastic limit in tension ($\sigma_{et}$) is equal to the elastic limit in compression ($\sigma_{ec}$). They are therefore both denoted simply as $\sigma_e$ (elastic limit). This is the case for steels.

- **Safety Factor**

The safety factor ranges from 1.5 to 2 for floors, from 2 to 3 for frameworks, and from 10 to 12 for elevators and cables.

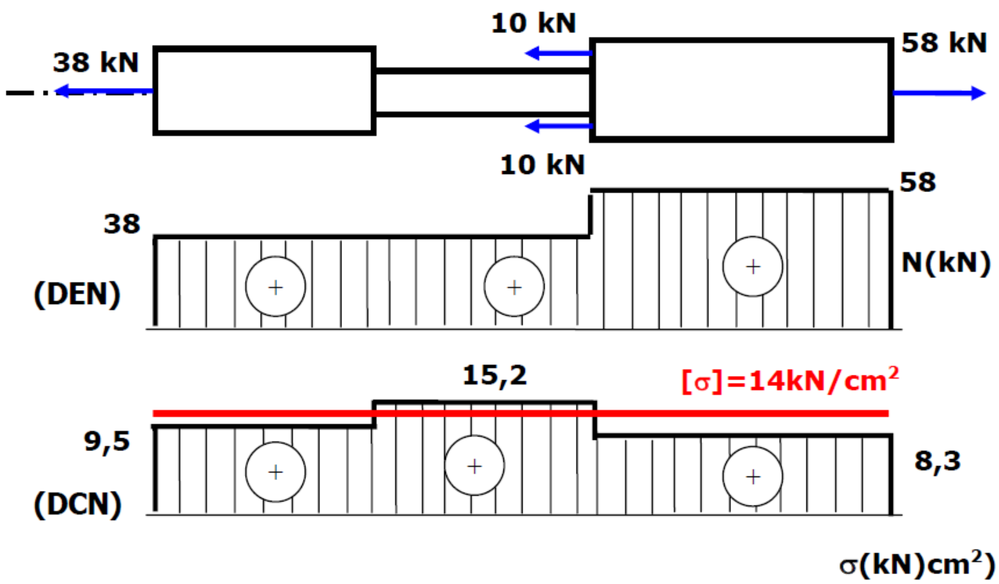
- **Example 3.2**

Verify the strength of the metal bar shown in the figure below, given that [σ] = 14 kN/cm$^2$.




- **Solution 3.2**

We draw the Normal Force Diagram and from it derive the Normal Stress Diagram, then we mark on it the value of the material's allowable stress:



We note that the maximum stress equals 15.2 kN/cm², which is greater than the allowable stress; therefore the bar does not withstand the tensile load (it fails in tension).

### 3.7. Law of Elastic Deformation

Consider a bar of initial length L subjected to a normal force N. A small portion of the bar of length dx undergoes a change in length du = Δ(dx) (see Fig. 3.4).

The longitudinal strain at the section located at abscissa **x** is defined as the dimensionless quantity:

$$\varepsilon = \frac{\Delta(dx)}{dx} \tag{3.5}$$

Where

$$\Delta(dx) = \varepsilon\, dx \tag{3.6}$$

On the other hand,

$$\varepsilon = \frac{\sigma}{E} = \frac{N}{ES} \tag{3.7}$$

Thus, Δ(dx) is equal to:

$$\Delta(dx) = \frac{N}{ES} dx \tag{3.8}$$

and the total deformation of the bar is therefore:

$$\Delta L = \int_0^L \Delta(dx) = \int_0^L \frac{N}{ES} dx \qquad 3.7$$

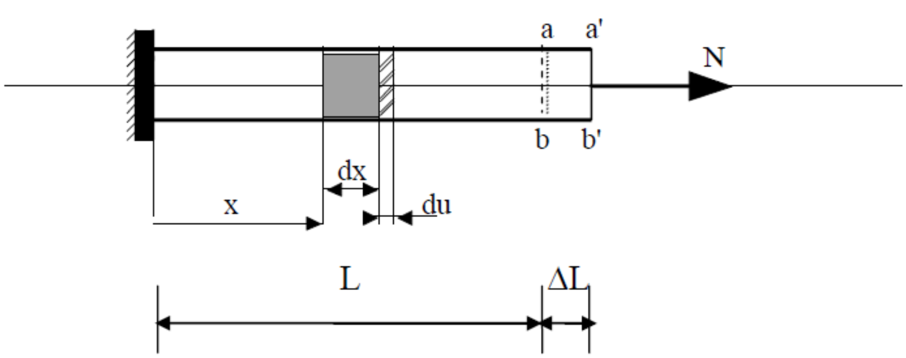
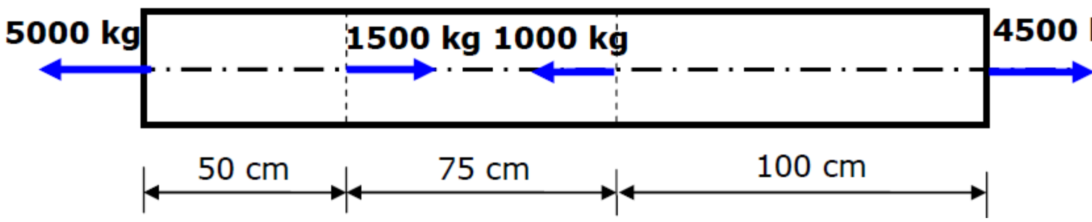



**Fig 3.4.** Linear Deformation

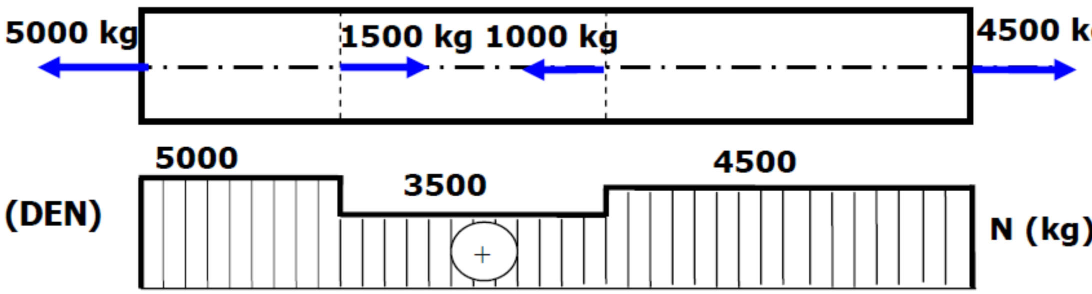
- **Example 3.3**

Determine the total elongation of the metal bar subjected to loading as shown in the figure below, knowing that the Young's modulus is $E=2.1\times10^6 kg/cm^2$. The cross-sectional area of the bar is constant and equal to $A = 5\ cm^2$




- **Solution 3.3**

The Normal Force Diagram is shown in the figure below:




$$\Delta L = \int_0^L \frac{N}{ES}dx = \int_0^{L_1} \frac{N_1}{ES_1}dx = \int_{L_1}^{L_1+L_2} \frac{N_2}{ES_2}dx = \int_{L_1+L_2}^{L} \frac{N_3}{ES_3}dx = \frac{N_1L_1}{ES_1} + \frac{N_2L_2}{ES_2} + \frac{N_3L_3}{ES_3} = \frac{1}{E}\sum_{i=1}^{n}\frac{N_iL_i}{S_i}$$

$$\Delta L = \frac{1}{2.1x10^6x5} = (5000x50 + 3500x75 + 4500x100)$$

Thus, the total elongation of the bar is:

$$\Delta L = 0.092\ Cm$$