University of Mila | Academic Year: 2025/2026
Institute of Nature and life Sciences | 1^{st} Year NLS, Semester 1
Department of NLS | Mathematics Statistics
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Exercises Serie N° 1

Exercise 1

Considered as functions of a real variable what is the domain of definition of each of the following functions ?

❶ $f(x) = \dfrac{1}{\sqrt{x+|x|}}$.

❷ $g(x) = \ln\left(\dfrac{2+x}{2-x}\right)$.

❸ $h(x) = \sqrt{x^2+3x-10}$.

Exercise 2

Study the parity of the following functions

- $f_1(x) = \dfrac{e^{2x}-1}{e^{2x}+1}$,
- $f_2(x) = \ln\left(x+\sqrt{x^2+1}\right)$,
- $f_3(x) = x^2+x$.

Exercise 3

❶ Show that

- The function f defined on $\mathbb{R}$ by: $f(x) = cos(2x) + sin(2x)$, periodic with period π.
- The function g defined on $]-1,+\infty[$ by: $g(x) = \frac{3x}{1+x}$, majorized by 3.
- The function h defined on $\mathbb{R}$ by: $h(x) = 3x^2 - 6x + 5$, minorized by 2.

❷ Study the monotonicity of the following function

$$E(x) = 3^{\frac{1}{x-1}},\ x \in]-\infty,1[\text{ and } x \in]1,+\infty[$$

Exercise 4

Calculate the following limits where they exist

❶ $\lim\limits_{x\to 0} \frac{e^x-1}{x^2+2x}$, ❷ $\lim\limits_{x\to 1} \frac{\sqrt{x^2-2x+1}}{x-1}$, ❸ $\lim\limits_{x\to +\infty} \frac{\sqrt{x+1}-\sqrt{2x}}{x-1}$, ❹ $\lim\limits_{x\to +\infty} e^{x-\sin(x)}$.

Exercise 5

❶ Study the continuity of the following functions

a) $f_1(x) = \begin{cases} e^x - 1 & if \quad x < 0, \\ 0 & if \quad x = 0, \\ x\ln(x) + x & if \quad x > 0, \end{cases}$

b) $f_2(x) = \begin{cases} a^{-\frac{1}{\sqrt{x}}} & if \quad x > 0,\ a > 0, \\ 0 & if \quad x \leqslant 0. \end{cases}$

❷Let a, b be real numbers, f a function defined by :

$$f(x) = \begin{cases} x\sin(x) - a\cos(x) & if \quad x < 0, \\ x + b & if \quad 0 \leqslant x \leqslant 3 \\ x^2 - 4 & if \quad x > 3. \end{cases}$$

Find values of a, b, that will make the function f continuous on $\mathbb{R}$.

Exercise 6

❶ Study the differentiability of the following functions at $x_0 = 0$

a) $g_1(x) = \begin{cases} x^2\cos(\frac{1}{x}), & if \quad x \neq 0, \\ 0, & if \quad x = 0. \end{cases}$ b) $g_2(x) = \begin{cases} 1 - e^x, & if \quad x \leqslant 0. \\ \frac{e^x-1}{e^x+1}, & if \quad x > 0. \end{cases}$

❷Let the function g defined on $\mathbb{R}_+$by :

$$g(x) = \begin{cases} ax^2 + bx + 1, & 0 \leq x \leq 1, \\ \sqrt{x}, & if \quad x > 1. \end{cases}$$

Determine the real numbers a, and b, so that g is differentiable on $\mathbb{R}_+$. Calculate $g'(x)$.

Exercise 7

Calculate the derivatives of the following functions

- $f_1(x) = \frac{\cos x}{\sin(x)+3}$,
- $f_2(x) = \ln\left(x^2 + e^{-x^2}\right)$,
- $f_3(x) = \sqrt{x^2 + 6x - 1}$,
- $f_4(x) = \sqrt[3]{x^3 + 2}$,
- $f_5(x) = (x+1)^4 e^{-x}$,
- $f_6(x) = \arctan\left(\frac{1+x}{1-x}\right)$.