

Chapter 3: Comparison of Two or More Means

Statistics Course — 3rd Year Food Quality and Safety

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1 Introduction

In many scientific studies, researchers need to compare the means of two or more groups to determine whether there are significant differences among them. For example:

- Comparing the average bacterial growth under different temperature conditions.
- Comparing the mean vitamin C content in juices from different brands.

The main statistical tools used are:

1. The **t-test** (for two means)
2. The **Analysis of Variance (ANOVA)** (for three or more means)

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2 Comparison of Two Means: The *t*-test

2.1 Case 1: Independent Samples

We compare two independent groups, e.g., two different production lines.

Hypotheses:

$$H_0 : \mu_1 = \mu_2 \quad (\text{no difference between means})$$

$$H_1 : \mu_1 \neq \mu_2 \quad (\text{means are significantly different})$$

Test Statistic:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where

$$S_p = \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}}$$

Decision Rule: Reject H_0 if $|t| > t_{\alpha/2, df}$

Example: Two laboratories measured the protein content of milk samples.

	n	\bar{X}	S
Lab A	10	3.5	0.4
Lab B	10	3.9	0.3

Compute:

$$t = \frac{3.5 - 3.9}{\sqrt{\frac{0.4^2}{10} + \frac{0.3^2}{10}}} = -2.53$$

At $\alpha = 0.05$, $t_{0.05,18} = 2.10$. Since $|t| > 2.10$, we **reject** H_0 . \rightarrow *The two labs have significantly different mean protein contents.*

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2.2 Case 2: Paired Samples (Dependent Samples)

Used when measurements are taken on the same units before and after treatment.

Example: Ten samples of fruit juice were tested before and after pasteurization.

We calculate the differences $d_i = X_{before} - X_{after}$ and use:

$$t = \frac{\bar{d}}{S_d/\sqrt{n}}$$

where S_d is the standard deviation of the differences.

3 Comparison of More Than Two Means: One-Way ANOVA

3.1 Purpose

To test whether three or more population means are equal:

$$H_0 : \mu_1 = \mu_2 = \dots = \mu_k$$

H_1 : At least one mean differs

3.2 Model

$$X_{ij} = \mu + \tau_i + \epsilon_{ij}$$

where:

- X_{ij} = observation j in group i
- μ = overall mean
- τ_i = effect of treatment i
- ϵ_{ij} = random error

3.3 Assumptions

1. Populations are normally distributed.
2. Populations have equal variances.
3. Samples are independent.

3.4 Test Statistic (F-test)

$$F = \frac{MS_{between}}{MS_{within}} = \frac{SS_{between}/(k-1)}{SS_{within}/(N-k)}$$

3.5 ANOVA Table Example

Source	SS	df	MS	F
Between Groups	SS_B	$k - 1$	$MS_B = SS_B / (k - 1)$	$F = MS_B / MS_W$
Within Groups	SS_W	$N - k$	$MS_W = SS_W / (N - k)$	
Total	SS_T	$N - 1$		

Decision: Reject H_0 if $F > F_{\alpha, k-1, N-k}$

Example: Comparing Three Types of Flour

Objective: Determine whether there is a significant difference in the mean protein content (%) among three types of flour: A, B, and C.

Step 1. Data

Flour Type	Observations (Protein %)
A	10.1, 10.5, 10.3, 10.4
B	10.8, 10.9, 11.0, 10.7
C	10.2, 10.3, 10.1, 10.4

Each group has $n = 4$ observations.

Step 2. Compute the Group Means

$$\bar{X}_A = \frac{10.1 + 10.5 + 10.3 + 10.4}{4} = 10.33$$

$$\bar{X}_B = \frac{10.8 + 10.9 + 11.0 + 10.7}{4} = 10.85$$

$$\bar{X}_C = \frac{10.2 + 10.3 + 10.1 + 10.4}{4} = 10.25$$

Step 3. Compute the Overall (Grand) Mean

$$\bar{X}_T = \frac{(10.1 + 10.5 + 10.3 + 10.4 + 10.8 + 10.9 + 11.0 + 10.7 + 10.2 + 10.3 + 10.1 + 10.4)}{12}$$

$$\bar{X}_T = \frac{126.7}{12} = 10.56$$

Step 4. Sum of Squares Between Groups (SSB)

$$SSB = n \sum (\bar{X}_i - \bar{X}_T)^2$$

$$SSB = 4[(10.33 - 10.56)^2 + (10.85 - 10.56)^2 + (10.25 - 10.56)^2]$$

$$SSB = 4[(-0.23)^2 + (0.29)^2 + (-0.31)^2] = 4(0.2331) = 0.9324$$

Step 5. Sum of Squares Within Groups (SSW)

$$SSW_A = (10.1 - 10.33)^2 + (10.5 - 10.33)^2 + (10.3 - 10.33)^2 + (10.4 - 10.33)^2 = 0.0756$$

$$SSW_B = (10.8 - 10.85)^2 + (10.9 - 10.85)^2 + (11.0 - 10.85)^2 + (10.7 - 10.85)^2 = 0.055$$

$$SSW_C = (10.2 - 10.25)^2 + (10.3 - 10.25)^2 + (10.1 - 10.25)^2 + (10.4 - 10.25)^2 = 0.05$$

$$SSW = 0.0756 + 0.055 + 0.05 = 0.1806$$

Step 6. Compute the Mean Squares

$$df_{\text{between}} = k - 1 = 3 - 1 = 2, \quad df_{\text{within}} = N - k = 12 - 3 = 9$$

$$MS_{\text{between}} = \frac{SSB}{df_{\text{between}}} = \frac{0.9324}{2} = 0.4662$$

$$MS_{\text{within}} = \frac{SSW}{df_{\text{within}}} = \frac{0.1806}{9} = 0.02007$$

Step 7. Compute the F-statistic

$$F = \frac{MS_{\text{between}}}{MS_{\text{within}}} = \frac{0.4662}{0.02007} = 8.64$$

Step 8. Decision Rule

At significance level $\alpha = 0.05$, the critical value is $F_{0.05,2,9} = 4.26$.

Since $F_{\text{calculated}} = 8.64 > 4.26$, we **reject the null hypothesis** H_0 .

Step 9. Conclusion

There is a statistically significant difference in the mean protein content among the three types of flour (A, B, and C). This implies that not all flours have the same mean protein percentage.

Summary Table

Source of Variation	SS	df	MS	F	F_{crit} (0.05)	Decision
Between Groups	0.9324	2	0.4662	8.64	4.26	Reject H_0
Within Groups	0.1806	9	0.0201			
Total	1.113	11				

4 Post-Hoc Comparisons (Multiple Comparison Tests)

When H_0 is rejected, we can use:

- **Tukey's HSD Test**
- **Bonferroni Correction**
- **Duncan Test**

These methods identify which specific groups differ significantly.

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5 Two-Way ANOVA (Brief Introduction)

If there are two factors (e.g., Temperature and pH), we can study:

- The main effect of each factor.
- The interaction effect between factors.

Model:

$$X_{ijk} = \mu + A_i + B_j + (AB)_{ij} + \epsilon_{ijk}$$

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6 Conclusion

- For two means → use **t-test**.
- For more than two means → use **ANOVA**.
- Check assumptions before applying tests.
- Use post-hoc tests for detailed comparisons.

End of Chapter 3 — Comparison of Two or More Means

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t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
Z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

F-table of Critical Values of $\alpha = 0.05$ for F(df1, df2)

DF1=	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.95	248.01	249.05	250.10	251.14	252.20	253.25	254.31
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00