In-Class Exercises n°02

| Heat Conduction – Part I |

Exercise 2.1

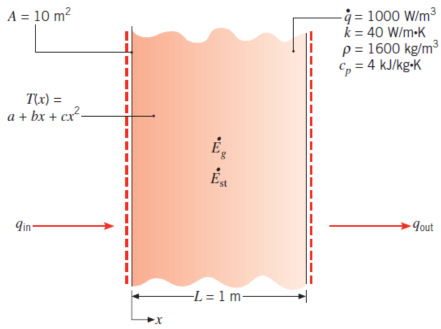
The temperature distribution across a wall **1 m thick** at a certain instant of time is given a

$$\mathbf{T}(x) = \mathbf{a} + \mathbf{b}x + \mathbf{c}x^2$$

Where: T is in degrees Celsius and **x** is in meters

While: **a = 900°C**, **b = −300°C/m**, and **c = −50°C/m²**

A uniform heat generation, $\dot{q}$ = **1000 W/m³**, is present in the wall of area **10 m²** having the properties: **ρ = 1600 kg/m³**, **k = 40 W/m·K**, and $\mathbf{c_p}$ **= 4 kJ/kg·K**



(Figure 2.1)

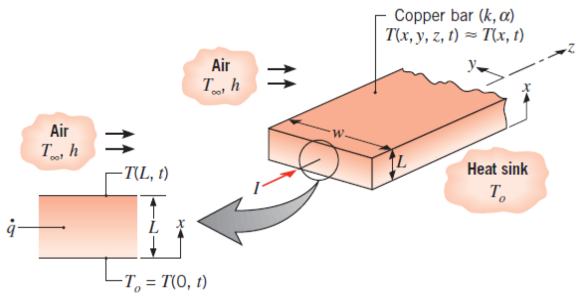
1. Determine the rate of heat transfer entering the wall (**x = 0**) and leaving the wall (**x = 1 m**).

2. Determine the rate of change of energy storage in the wall.

3. Determine the time rate of temperature change at **x = 0, 0.25**, and **0.5 m**.

Exercise 2.2

A long copper bar of rectangular cross section, whose width w is much greater than its thickness **L**, is maintained in contact with a heat sink at its lower surface, and the temperature throughout the bar is approximately equal to that of the sink, $\mathbf{T_o}$. Suddenly, an electric current is passed through the bar and an airstream of temperature $\mathbf{T_\infty}$ is passed over the top surface, while the bottom surface continues to be maintained at $\mathbf{T_o}$.

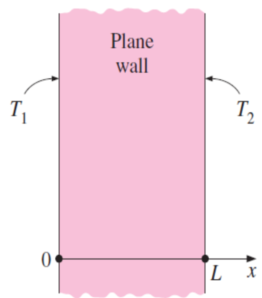


(Figure 2.2)

Obtain the differential equation and the boundary and initial conditions that could be solved to determine the temperature as a function of position and time in the bar.

Exercise 2.3

Consider a large plane wall of thickness **L = 0.2 m**, thermal conductivity **k = 1.2 W/m·°C**, and surface area **A = 15 m²**. The two sides of the wall are maintained at constant temperatures of $\mathbf{T_1}$ **= 120 °C** and $\mathbf{T_2}$ **= 50 °C**, respectively, as shown in **(Figure 2.3)**.



(Figure 2.3)

Determine

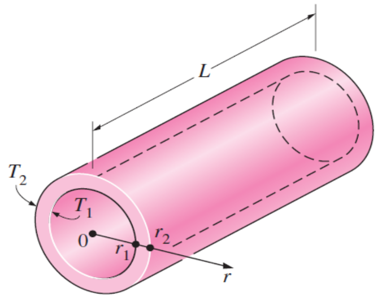
a) the variation of temperature within the wall and the value of temperature at x = **0.1 m**

b) the rate of heat conduction through the wall under steady conditions.

Exercise 2.4

Consider a steam pipe of length **L = 20 m**, inner radius $\mathbf{r_1 = 6}$ **cm**, outer radius $\mathbf{r_2 = 8}$ **cm**, and thermal conductivity **k = 20 W/m·°C**, as shown in **Figure 2–4**. The inner and outer surfaces of the pipe are maintained at average temperatures of $\mathbf{T_1}$ **= 150 °C** and $\mathbf{T_2}$ **= 60 °C**, respectively.

Obtain a general relation for the temperature distribution inside the pipe under steady conditions, and determine the rate of heat loss from the steam through the pipe.

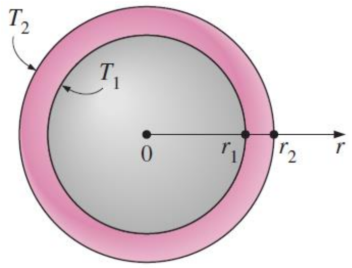


(Figure 2.4)

Exercise 2.5

Consider a spherical container of inner radius $\mathbf{r_1 = 8}$ **cm**, outer radius $\mathbf{r_2 = 10}$ **cm**, and thermal conductivity **k = 45 W/m·°C**, as shown in **Figure 2–5**. The inner and outer surfaces of the container are maintained at constant temperatures of $\mathbf{T_1}$ **= 200 °C** and $\mathbf{T_2}$ **= 80 °C**, respectively, as a result of some chemical reactions occurring inside.

Obtain a general relation for the temperature distribution inside the shell under steady conditions, and determine the rate of heat loss from the container.



(Figure 2.5)

Exercise 2.6

Consider, for each situation, a medium in which the heat conduction equation is given in its simplest form as

Situation 1	Situation 2	Situation 3
$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$	$\frac{1}{r}\frac{d}{dr}\left(rk\frac{dT}{dr}\right) + \dot{g} = 0$	$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$

a) Is heat transfer steady or transient?
b) Is heat transfer one-, two-, or three-dimensional?
c) Is there heat generation in the medium?
d) Is the thermal conductivity of the medium constant or variable?

Exercise 2.7

Beginning with a differential control volume in the form of a cylindrical shell, derive the heat diffusion equation for a one-dimensional, cylindrical, radial coordinate system with internal heat generation. Compare your result with Equation (**Eq.02**).

Exercise 2.8

Beginning with a differential control volume in the form of a spherical shell, derive the heat diffusion equation for a one-dimensional, spherical, radial coordinate system with internal heat generation. Compare your result with Equation (**Eq.03**).