

Bézier Surfaces:

The most commonly used approach to defining a Bézier surface is that of the tensor product. According to this approach, a surface can be defined as the set of points of a curve moving and transforming simultaneously (according to Bézier).

If we consider two curves in space defined as follows:

$$F(u) = \sum_i N_i(u) \mathbf{b}_i \quad \text{et} \quad G(v) = \sum_j M_j(v) \mathbf{d}_j$$

Their tensor product is defined as follows:

$$S(u, v) := \sum_i \sum_j N_i(u) M_j(v) \mathbf{c}_{i,j}$$

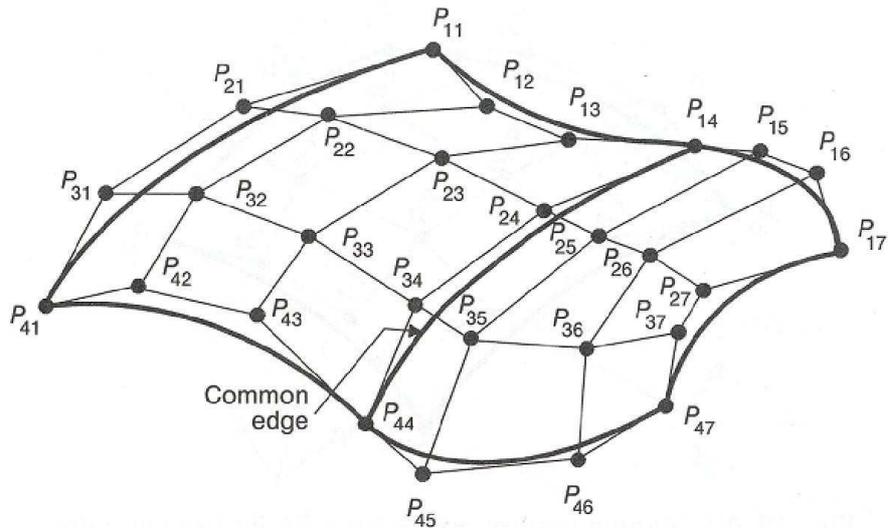
For the case of a Bézier surface:

$$S(u, v) := \sum_{i=0}^n \sum_{j=0}^m B_i^n(u) B_j^m(v) \mathbf{b}_{i,j}$$

If we take this definition again, we can notice that:

$$S(u, v) = \sum_i \sum_j N_i(u) M_j(v) \mathbf{c}_{i,j} = \sum_i N_i(u) \left(\sum_j M_j(v) \mathbf{c}_{i,j} \right) = \sum_j M_j(v) \left(\sum_i N_i(u) \mathbf{c}_{i,j} \right)$$

For a fixed value of u we obtain a curve in the direction v and vis versa. Se which means that the surface obtained by tensor product is a curve of curves.



Evaluation of a Bézier surface:

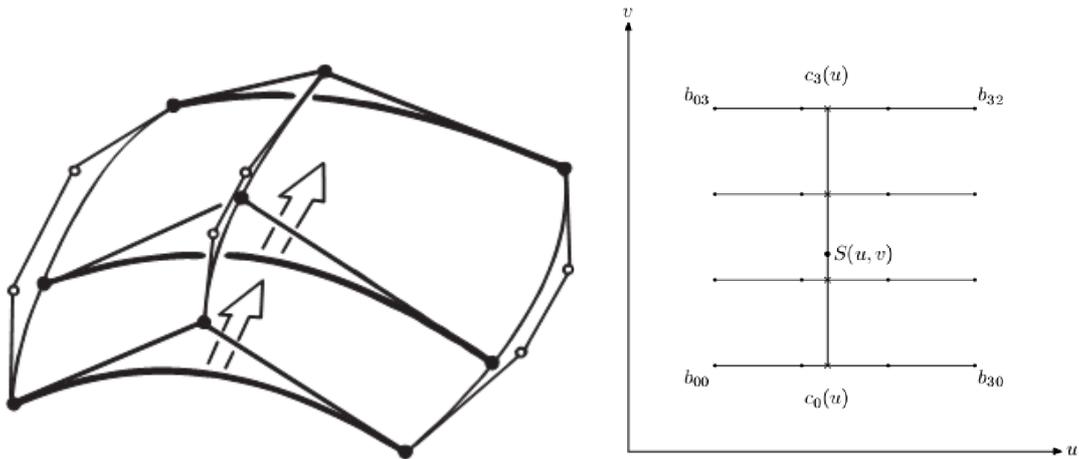
1. With the tensor product approach:

The procedure consists of:

1. Apply the de Casteljau algorithm for a curve in one direction (n+1) times.
2. Apply the de Casteljau algorithm for a curve in the other direction only once.

For a better order of evaluation (for a minimum of arithmetic operation):

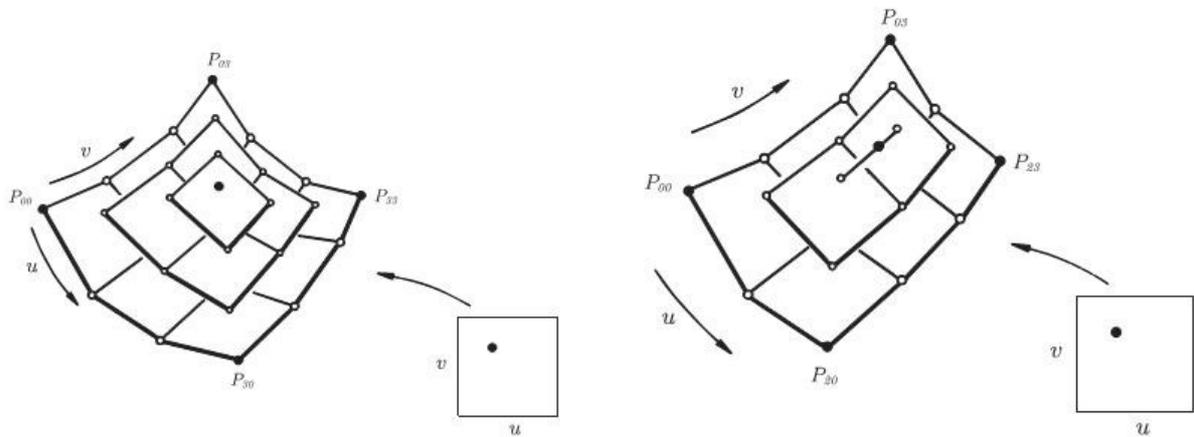
3. If $m > n$ start with u then v (see figure)
4. If $n > m$ start with v then u



1) With the direct approach of de Casteljau:

It is an adaptation of the de Casteljau algorithm presented for Bézier curves for the construction of Bézier surfaces. If we consider a set of control points consisting of $n \times p$ points, then the point of the Bézier surface associated with the parameters (u, v) is constructed as follows. For all the tiles of the surface, we calculate the bilinear interpolation corresponding to the point (u, v) ; This makes it possible to obtain a set of new points which in turn define a set of tiles (in the first step: $n-1 \times p-1$).

The process is then repeated iteratively until only one point is obtained.



Continuity of composed surface:

For simplicity, it is assumed that the surfaces have the same number of control points (i.e. $n \times p$) and that the surfaces are fitted on a side with the same number of control points.

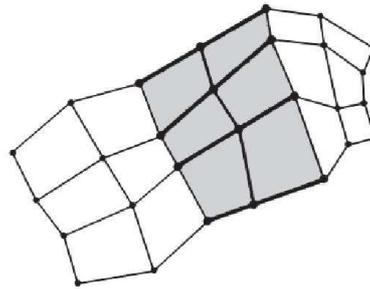
For the continuity C0, the Bézier curves on the common edge must coincide:

$$B(0, v) = B'(1, v) \text{ pour tout } v \in [0, 1]$$

Since these two Bézier curves have the same number of control points, they coincide when they are confused. The continuity C0 therefore implies that the Bézier curve B_i on the surface B is extended by the Bézier curve B_i on the surface B' (for all i). For checkpoints, this implies equality $P_{0j} = P'_{nj}$

Where P_{ij} et P'_{ij} are the control points for surfaces B and B'

For higher-order continuities, it is therefore sufficient to consider the degree of continuity of the $B_i \cup B'_i$ curves for any i .



Continuité C^1 entre deux surfaces de Bézier.

Surface B-spline :

A B-spline surface of degree (d_1, d_2) is the parameterized surface defined by the following tensor product:

$$P(u,v) = \sum_{i=0}^{n_1} \sum_{j=0}^{n_2} N_i^{d_1}(u) N_j^{d_2}(v) P_{ij}$$

où $P_{ij} \in \mathbb{R}^d$, ($d = 2$ ou $d = 3$).

To construct a B-spline surface of degrees (d_1, d_2) from the (n_1+1) and (n_2+1) control points P_i , it is necessary to give $(m_1+1)(m_2+1)$ nodes where $m_1=n_1+d_1+1$ and $m_2=n_2+d_2+1$, allowing to define the basic functions.

Note that $P(u,v)$ can be written as:

$$P(u,v) = \sum_{i=0}^{n_1} N_i^{d_1}(u) \left(\sum_{j=0}^{n_2} N_j^{d_2}(v) P_{ij} \right) = \sum_{i=0}^{n_1} N_i^{d_1}(u) \tilde{P}_i(v)$$

All properties on B-spline curves apply to B-spline surfaces in each direction.

Algorithm 4 Évaluation d'une surface en un point (u,v)

Entrée: $d_1, d_2, (u,v), U = [u_0, \dots, u_{m_1}], V = [v_0, \dots, v_{m_2}], P = \begin{bmatrix} P_{00} & \dots & P_{0n_2} \\ \vdots & \ddots & \vdots \\ P_{n_1 0} & \dots & P_{n_1 n_2} \end{bmatrix}$

Sortie: $P(u,v)$

Algorithme:

Évaluer les $n_2 + 1$ points $\tilde{P}_i(v) =$

$\sum_{j=0}^{n_2} N_j^{d_2}(v) P_{ij}$ à l'aide de l'algorithme de De Boor;

Évaluer $P(u,v) = \sum_{i=0}^{n_1} N_i^{d_1}(u) \tilde{P}_i(v)$ à l'aide de l'algorithme de De Boor;

NURBS surfaces:

As with Bézier surfaces or B-spline surfaces, NURBS surfaces can be obtained through the tensor product of two NURBS curves, for this purpose we use parameters u and v with two indices i and j .

$$C(t) = \sum_{i=0}^n \sum_{j=0}^l R_{i,j}(u,v) P_{i,j}$$

Avec

$$R_{i,j}(u,v) = \frac{N_{i,k}(u) N_{j,l}(v) w_{i,j}}{\sum_{p=1}^n \sum_{q=1}^m N_{p,k}(u) N_{q,l}(v) w_{p,q}}$$

As rational basis fonctions.