Democratic and Popular Republic of Algeria

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Analysis 2

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Exercises Serie N° 3

Note: questions marked (∗) left to the students

Exercise 1

Solve the following first-order differential equations:

1. $y' - 2xy = (1-2x)e^x, \quad y(0) = 5;$
2. $2x + yy' = 0, \quad y(1) = 1.$
3. $xy' + (1+x)y = 0, \quad y(1) = 1;$
4. $(4-x^2)yy' = 2(1+y^2).$
5. $y' - 2xy = e^{x^2}\sin x, \quad y(0) = 1;$
6. $y' - y\cos x = \cos x, \quad y(0) = 0 \cdots (*)$
7. $xy' + y = x, \quad y(2) = 0;$
8. $y' - 2y = -\dfrac{2}{1+e^{-2x}}, \quad y(0) = 2.$
9. $y' - \left(2x - \dfrac{1}{x}\right)y = 1, \quad on \quad]0, +\infty[;$
10. $(1+x)y' + xy = x^2 - x + 1, \quad y(1) = 1 \cdots (*)$
11. $xy' + 3y = x^2y^2;$
12. $y' + 2xy = -xy^4.$
13. $xy' - y = y^2 \ln x \cdots (*)$

Exercise 2

Consider the following differential equation:

$$(x^2 - 3x + 2)y' - y^2 + 3xy = 4x^2 - 6x + 4 \cdots\cdots (E)$$

1. *Show that* $y = 2x$ *is a particular solution of the equation* (E).
2. *Solve the equation* (E) *on* $]2, +\infty[$.

Exercise 3

Solve the following second-order differential equations:

1. $y'' + y' - 6y = 4e^x, \qquad y(0) = 1, \; y'(0) = -22.$
2. $y'' - 3y' + 2y = (1-2x)e^x.$
3. $y'' - 2y' + 2y = 5\cos x, \qquad y(0) = 1, \; y'\left(\dfrac{\pi}{2}\right) = -2\left(e^{\frac{\pi}{2}} + 1\right).$
4. $y'' + 4y = 2\sin x \cos x.$
5. $y'' - 4y' + 4y = xe^{2x}, \qquad y(0) = 1, \; y'(0) = 4.$
6. $y'' + 4y = e^{3x}\cos(2x).$
7. $y'' - 8y' + 15y = 15x^2 - 16x + 17, \qquad y(0) = 3, \; y'(1) = 2(1+e^3) \cdots\cdots (*)$

Exercise 4

Consider the equation: $y'' + 2y' + 4y = xe^x \cdots\cdots (E)$

1. *Solve the homogeneous differential equation associated with* (E).
2. *Find a particular solution of* (E)*, then give the set of all solutions of* (E).
3. *Determine the unique solution* h *of* (E) *satisfying* $h(0) = 1$*, and* $h(1) = 0$.