Abdelhafid Bousouf University Center, Mila
Institute of Mathematics and Computer Science
Second year in Mathematics and Applied Mathematics, 2025/2026

Module : *Algebra 3*
Course leader : *Dr. Haddad.N*

WORKSHEET 1 (PREREQUISITES)

Exercise 1

Let f and g be the applications defined by :

$$f: \begin{array}{ll} \mathbb{R}^2 & \to \mathbb{R}^2 \\ (x,y) & \mapsto (2x-4y, x-2y) \end{array}$$

$$g: \begin{array}{ll} \mathbb{R}_3[X] & \to \mathbb{R}_3[X] \\ P & \mapsto (1+X)P' \end{array}$$

1. Show that f and g are endomorphisms.
2. Give the matrix M_f of f in the canonical basis of $\mathbb{R}^2$.
3. Give the matrix M_g of g in the canonical basis of $\mathbb{R}_3[X]$.

Exercise 2

Consider the application

$$f: \begin{array}{ll} \mathbb{R}_2[X] & \to \mathbb{R}_2[X] \\ P & \mapsto (1+X)P' \end{array}$$

1. Show that f is an endomorphism.
2. Determine M, the matrix associated with f in the canonical basis B of $\mathbb{R}_2[X]$.
3. Show that $B' = \{1, (1+X), (1+X)^2\}$ is a basis of $\mathbb{R}_2[X]$.
4. Determine M', the matrix associated with f in the basis B'.
5. Determine P, the change of basis matrix from B to B'.
6. Show that P is invertible and find P^{-1}.
7. Deduce the matrix M' from the matrices P, P^{-1}, and M.

Exercise 3

Calculate the determinants of the following matrices :

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 3 & 5 \\ 4 & 1 & 3 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 123 & -3 & 0 & 0 \\ 0 & -79 & 61 & 0 \\ 85 & -93 & 0 & 4 \end{pmatrix}, C = \begin{pmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \\ 3 & 5 & 7 & 9 \\ 4 & 6 & 8 & 10 \end{pmatrix},$$

$$D = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 8 & 7 & 6 & 5 \\ 9 & 10 & 11 & 12 \\ 16 & 15 & 14 & 13 \end{pmatrix}, E = \begin{pmatrix} 1 & a & b & a \\ 1 & b & 1 & b \\ 1 & 1 & 1 & a \\ 1 & 1 & a & b \end{pmatrix}, a, b \in \mathbb{R}.$$

Exercise 4

Consider the matrix in $M_3(\mathbb{R})$:

$$M_\alpha = \begin{pmatrix} 4 & -1 & 3 \\ 2 & 2 & 1 \\ \alpha & 1 & \alpha - 2 \end{pmatrix}.$$

1. Calculate the determinant of M_α and deduce the values of α for which M_α is invertible.
2. Calculate M_1^{-1}, the inverse of the matrix M_1.

Exercise 5

Consider the application

$$\begin{aligned} f : \quad & \mathbb{R}^3 \quad \to \mathbb{R}^4 \\ & (x, y, z) \mapsto f(x, y, z) = (x + z, y - x, z + y, x + y + 2z) \end{aligned}$$

1. Show that f is a linear application and determine M, the matrix associated to f with respect to B, the canonical basis of $\mathbb{R}^3$, and B', the canonical basis of $\mathbb{R}^4$.
2. Determine a basis of $\text{Im}(f)$. Deduce the rank of f.
3. Determine a basis of $\ker(f)$.
4. Is the application f injective ? Surjective ?

Exercise 6

Let

$$\begin{aligned} f : \ & \mathbb{R}[X] \to \mathbb{R}[X] \\ & P \mapsto P - XP' \end{aligned}$$

1. Show that f is endomorphism.
2. Determine the Kernel and the Image of f.