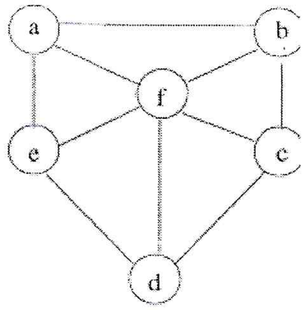
First name: .... Last name: .... Group: ....

2nd Year Computer Science, Module Supervisor: Dr. A. Merabet, Duration: 90 minutes, 2024/2025

Make-up Exam - Graph Theory

Documents, calculators and mobile phones are not allowed.

Exercise 1: 6,5



1,5 **Q1)** Find the limits of a chromatic number in the following graph without using the algorithm: $? \leq \gamma(G) \leq ?$

(Justify your answer)

1 **Q2)** Give the number of edges in a complete graph of order n?

1 **Q3)** Among K2, K3, K4, and K5, which ones are planar?

1 **Q4)** Prove that: $\sum_{v \in V} d(v) = 2\,|E|$

1 **Q5)** Prove that in a complete graph of order n, each vertex is of degree n-1:

1 **Q6)** Propose an algorithm to convert an adjacency matrix to an incidence matrix.

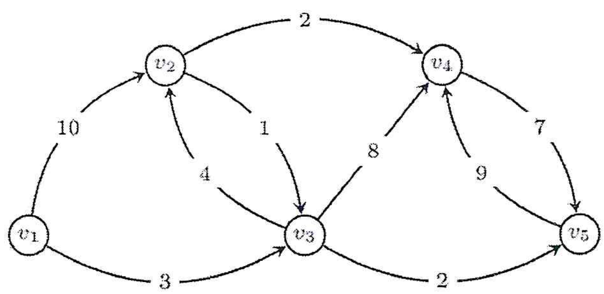
Exercise 2: 4,5

We want to set up a network of 5 machines (named 1 to 5) operating on limited channels. The machines must operate under the following constraints: M1 cannot operate at the same time as M2, M3, or M4. M4 cannot operate at the same time as M3 or M5.

A. In graph theory, what is the formal problem being described (justify your answer)?

B. Find the maximum number of machines that can operate simultaneously (detailed answer)?

Exercise 3: 09



1 **1.** Represent this graph G using an adjacency list.

0,5 **2.** Is G strongly connected? Why?

1 **3.** Find all strongly connected components.

0,5 **4.** Find the condensed graph Cc of G.

0,5 **5.** Find the graph complement $\bar{C}$ of Cc.

1,5 **6.** Using Solin Algorithm, find the minimum spanning tree of G.

3,25 **7.** Using Bellman-Kalaba Algorithm, find the shortest path between v1 and v5.

0,75 **8.** Deduce the shortest path between v1 and v2, between v1 and v3, between v1 and v4.

Make-up Exam 2024 — 2025

THG Rattrapage Solution

Exercise 01:

Q1) $3 \leq \delta(G) \leq 4$

- the chromatic number is always bigger than the bigest clique $C = \{a, b, e\}$ of order 3
- the chromatic number is always less than the number of maximum stables $S1 = \{a, c\}$, $S2 = \{b, e\}$, $S3 = \{d\}$, $S4 = \{f\}$.

Q2) number of edges in a complete graph is: $\frac{n(n-1)}{2}$

Q3) only K2, K3, K4 are planar

Q4) each edge (v_1, v_2) is counted twice when calculating the sum of degrees: one on the v1 side in the calculation of $d(v_1)$, and a second time on v_2 side in the calculation of $d(v_2)$

Q5) $\sum_{v \in V} d(v) = 2|E|$

in a complete graph all degrees are the same so: $\sum_{v \in V} d(v) = n.d$

$n.d = 2|E| = 2 . \frac{n.(n-1)}{2}$

$\boxed{d = n - 1}$

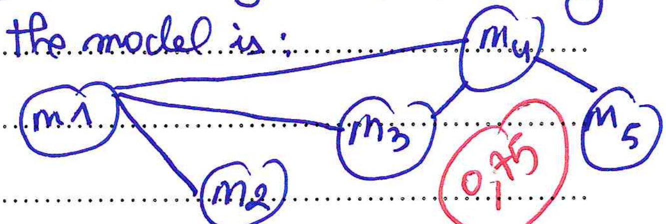
Q6) adjacency matrix to incidence matrix

```
edge ← 1
for i going from 1 to n do
    for j going from 1 to n do
        if Mat[i,j] = 1 then
            Incidence[i, edge] = 1
            Incidence[j, edge] = -1
        endif
    endfor
endfor
```

Exercise 02:

a) this problem is related to graph coloring in graph theory 0,75

the model is:



2 connected machine in the graph cannot work together in the same configuration ⇒ when coloring a graph 2 connected machines cannot get the same color ⇒ we look to machines with same color ⇒ especialy the color with the maximum machines 0,75

(1)

b) Let's color the graph

L	m_1	m_4	m_3	m_2	m_5
	3	3	2	1	1

0,5

current color = 0

$L = \{\cancel{m_1}, m_4, m_3, m_2, \cancel{m_5}\}$

cc = 1

0,25 $\boxed{S \equiv m_1 \equiv cc_1}$ $V = \{m_2, m_3, m_4\}$

0,25 $\boxed{x \equiv m_5 \equiv cc_1}$ $V = \{m_2, m_3, m_4\}$

$L = \{\cancel{m_4}, m_3, \cancel{m_2}\}$

cc = 2

0,25 $\boxed{S \equiv m_4 \equiv cc_2}$ $V = \{m_3\}$

0,25 $\boxed{x \equiv m_2 \equiv cc_2}$ $V = \{m_3\}$

$L = \{m_3\}$

cc = 3

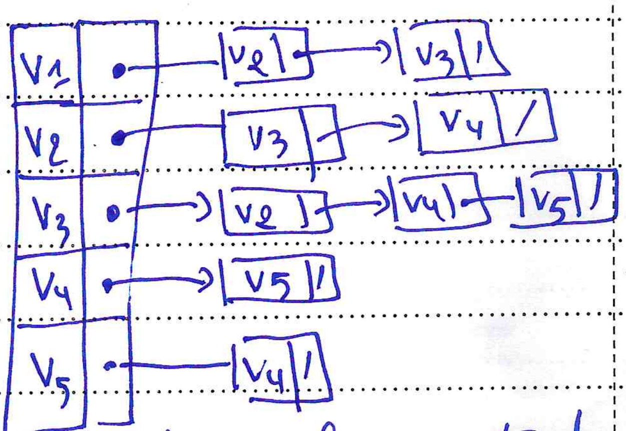
0,25 $\boxed{S \equiv m_3 \equiv cc_3}$ $V = \emptyset$

$L = \emptyset$

0,5 $cc_1 = \{m_1, m_5\}$, $cc_2 = \{m_4, m_2\}$ $cc_3 = \{m_3\}$ and those are the machines that can work together.

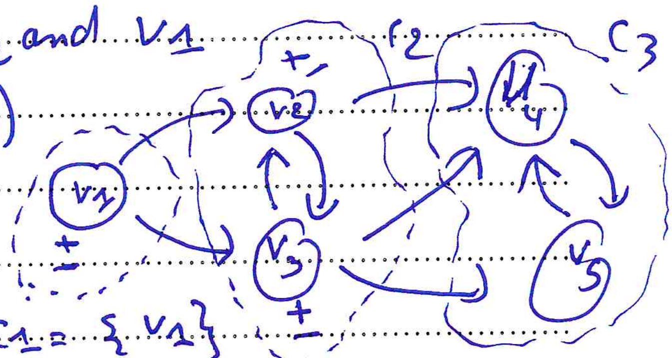
Exercise 03:

Q1) 1



Q2) G is not strongly connected because there is no path between v_2 and v_1. 0,5

Q3)

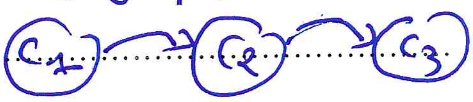


$C_1 = \{v_1\}$

$C_2 = \{v_2, v_3\}$

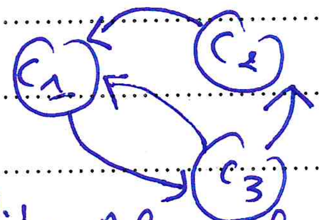
$C_3 = \{v_4, v_5\}$ 1

Q4) the C_c graph



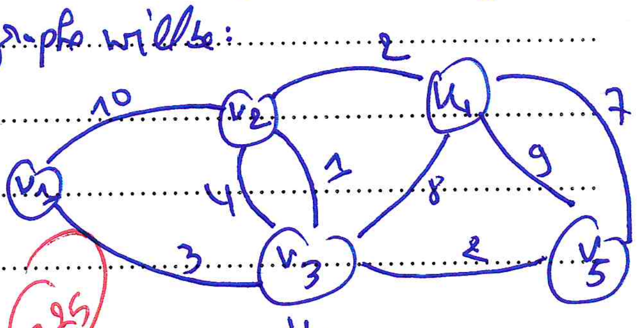
0,5

Q5)

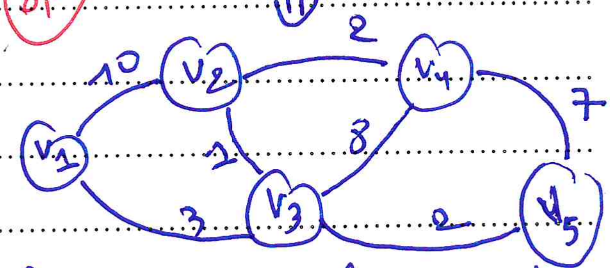


0,5

Q6) Sollin's Algorithm runs on undirected graphs only so the graph will be:



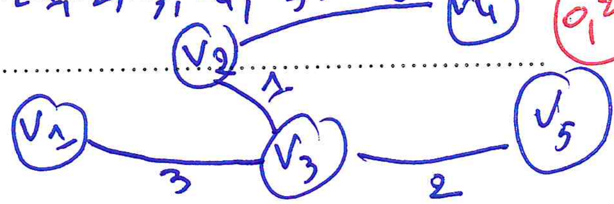
0,25



after removing loops and parallel edges

$E' = \emptyset$

$V' = \{v_1, v_2, v_3, v_4, v_5\}$



0,25

0,25 $\boxed{S \equiv v_1}$, $e_1 = (v_1, v_3)$, $E' = \{e_1\}$

$V' = \{v_2, v_4, v_5\}$

0,25 $\boxed{S \equiv v_2}$, $e_2 = (v_2, v_3)$, $E' = \{e_1, e_2\}$

$V' = \{v_4, v_5\}$

0,25 $\boxed{S \equiv v_4}$, $e_3 = (v_4, v_2)$, $E' = \{e_1, e_2, e_3\}$

$V' = \{v_5\}$

0,25 $\boxed{S \equiv v_5}$, $e_4 = (v_5, v_2)$, $E' = \{e_1, e_2, e_3, e_4\}$

$V' = \emptyset \Rightarrow$ end

G is reduced to one vertex

$\Rightarrow$ end

Q7) Bellman Kalaba :

L	$\lambda^k(v_1)$	$\lambda^k(v_2)$	$\lambda^k(v_3)$	$\lambda^k(v_4)$	$\lambda^k(v_5)$
k=0	0	∞	∞	∞	∞
k=1	0	10	3	11	5
k=2	0	7	3	9	5
k=3	0	7	3	9	5

1 1 saisie calcul 1

$$\lambda^k(x) = \min\left(\lambda^{k-1}(x), \min_{y \in \Gamma^{-1}(x)} \left(\lambda^{k-1}(y) + M(y,x)\right)\right)$$

$\lambda^1(v_2) = \min(\infty, \min(\lambda^0(v_1) + M(v_1,v_2), \lambda^0(v_3) + M(v_3,v_2)))$

$= \min(\infty, 0+10, \infty+4)$

$= \boxed{10}$ → 0,25

$\lambda^1(v_3) = \min(\infty, \min(\lambda^0(v_2) + M(v_2,v_3), \lambda^0(v_1) + M(v_1,v_3)))$

$= \min(\infty, 10+1, 0+3) = \boxed{3}$ → 0,25

$\lambda^1(v_4) = \min(\infty, \min(10+2, 3+8, \infty+9)) = \boxed{11}$ → 0,25

$\lambda^1(v_5) = \min(\infty, \min(3+2, 11+7)) = \boxed{5}$ → 0,25

3

$\lambda^2(v_2) = \min(10, \min(0+10, 3+4)) = \boxed{7}$ 0,25

$\lambda^2(v_3) = \min(3, \min(0+3, 7+1)) = \boxed{3}$ 0,25

$\lambda^2(v_4) = \min(11, \min(7+2, 3+8, 5+9)) = \boxed{9}$ 0,25

$\lambda^2(v_5) = \min(5, \min(3+2, 9+7)) = \boxed{5}$ 0,25

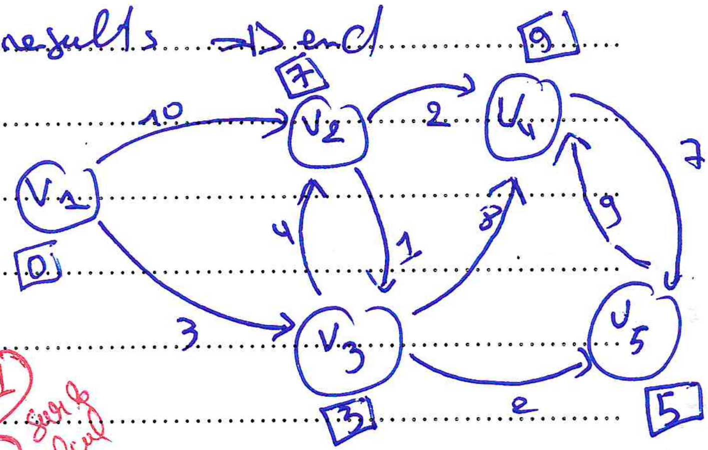
$\lambda^3(v_2) = \min(7, \min(0+10, 3+4)) = \boxed{7}$ 0,25

$\lambda^3(v_3) = \min(3, \min(0+3, 7+1)) = \boxed{3}$ 0,25

$\lambda^3(v_4) = \min(9, \min(7+2, 3+8, 5+9)) = \boxed{9}$ 0,25

$\lambda^3(v_5) = \min(5, \min(3+2, 9+7)) = \boxed{5}$ 0,25

k=2 and k=3 return the same results $\Rightarrow$ end



The shortest path between

$v_1, v_5 \Rightarrow \{(v_1, v_3), (v_3, v_5)\} \Rightarrow 5$ 0,25

$v_1, v_4 \Rightarrow \{(v_1, v_3), (v_3, v_2), (v_2, v_4)\} \Rightarrow 9$ 0,25

$v_1, v_3 \Rightarrow \{(v_1, v_3)\} \Rightarrow 3$ 0,25

$v_1, v_2 \Rightarrow \{(v_1, v_3), (v_3, v_2)\} \Rightarrow 7$ 0,25