MATHEMATICS II
WORK SHEET 03

Exercise 1. Find the next anti-derivatives:

① $\int x(1-x^2)^3\,\mathrm{d}x$ ③ $\int \frac{\ln x}{x}\,\mathrm{d}x$ ⑤ $\int \frac{1}{x\ln x}\,\mathrm{d}x.$

② $\int x\sqrt{1+4x^2}\,\mathrm{d}x$ ④ $\int xe^{-3x^2+7}\,\mathrm{d}x$

Exercise 2. Use substitution method to find the next anti-derivatives, then conclude there definite integrals.

① $\int \sqrt{\sin x}\cos x\,\mathrm{d}x, \quad \int_{\frac{\pi}{2}}^{\pi} \sqrt{\sin x}\cos x\,\mathrm{d}x.$

② $\int \frac{x}{x^2+1}\,\mathrm{d}x, \quad \int_0^1 \frac{x}{x^2+1}\,\mathrm{d}x.$

Exercise 3. Find the next integrals using integration by parts.

① $\int x\sin x\,\mathrm{d}x.$

② $\int \ln x\,\mathrm{d}x.$

Exercise 4. Calculate the next integrals.

① $I = \int_{-3}^{1} |x+1|\,\mathrm{d}x.$ ③ $K = \int_2^0 \sqrt{|x-1|}\,\mathrm{d}x.$

② $J = \int_0^3 |x^2-3x+2|\,\mathrm{d}x.$

Exercise 5. Let f be a real function and $a \in \mathbb{R}$. Prove that

① If f is even, then $\int_{-a}^{a} f(x)\,\mathrm{d}x = 0.$

② If f is odd, then $\int_{-a}^{a} f(x)\,\mathrm{d}x = 2\int_0^a f(x)\,\mathrm{d}x.$